Fiscal and monetary policy in New Keynesian DSGE models

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Declaration

I declare that this thesis does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any university and that to the best of my knowledge does not contain any materials previously published or written by another person except where due reference is made in the text. The views expressed in this thesis are those of the author only, and do not necessarily reflect the institutions that the author is currently or previous affiliated with.

Petros Varthalitis
July 2014
Abstract

This thesis is about monetary and fiscal policy in New Keynesian DSGE models. Chapter 2 presents the baseline New Keynesian DSGE model. Monetary policy is in the form of a simple interest rate Taylor-type policy rule, while fiscal policy is exogenous. Chapter 3 extends the model of Chapter 2 to include fiscal policy. Now, both monetary and fiscal policy are allowed to follow feedback rules. Chapter 4 sets up a New Keynesian model of a semi-small open economy with sovereign risk premia. Finally, Chapter 5 builds a New Keynesian DSGE model consisting of two heterogeneous countries participating in a monetary union. Throughout most of the thesis, policy is conducted via "simple", "implementable" and "optimized" feedback policy rules. Using such rules, the aim of policy is twofold: firstly, it aims to stabilize the economy when the latter is hit by shocks; secondly, it aims to improve the economy’s resource allocation.
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1 Introduction

This thesis is about monetary and fiscal policy in New Keynesian DSGE models. It studies both closed and open economies.

New Keynesian models (see Woodford, 2003, and Gali, 2008) depart from the standard neo-classical growth model in two key features. Firstly, the New Keynesian model allows the firm sector to operate under monopolistic competition, thus firms solve a profit-maximization problem by also choosing the price of their product. Secondly, firms face a nominal fixity, e.g. a Calvo-type nominal fixity which means that, in each time period, only a fraction of firms can reset their prices, while the remaining firms just set their previous period price. This fixity opens the door to a real role for monetary and exchange rate policy.

Throughout the thesis, policy is conducted via simple and implementable policy rules. Using such rules, the aim of policy is (at least) twofold. Firstly, it aims to stabilize the economy when the latter is hit by exogenous shocks. Secondly, it aims to improve the resource allocation of the economy.

We use numerical methods to solve, calibrate and simulate the models below. We perform both positive and normative analysis. Regarding positive analysis, we use standard methods in the recent literature (see Uhlig, 1997, Klein, 2000, Schmitt-Grohé and Uribe, 2004, or Dynare) to solve and simulate the models, find simulated moments and compare them to the actual data. Regarding normative analysis, we mainly build upon the influential work of Schmitt-Grohé and Uribe (2004, 2005 and 2007), by computing the so-called "optimized" policy rules. This is because we do not want our results to be driven by ad-hoc assumptions and differences in feedback policy coefficients across different policy regimes.

The thesis is structured as follows. Chapter 2 presents the baseline New Keynesian DSGE model. We start with the simplest possible version of the NKM, thus, monetary policy is in the form of a simple interest rate policy rule, i.e. a Taylor rule, while fiscal policy is exogenous. In this chapter, we experiment with several popular specifications of Taylor-type rules analyzing their effect on output and inflation based on simulated moments of the model.

Chapter 3 extends the model of the previous chapter to include fiscal policy. Now, both monetary and fiscal policy are allowed to follow feedback rules. In particular, we specify feedback policy rules for the nominal interest rate, the output share of public spending, the tax rate on consumption, the tax rate on labour income and the tax rate on capital income. We allow these policy instruments to react to several macroeconomic indicators, i.e. the nominal interest rate can react to inflation and the output gap, while the four fiscal policy instruments can react to the output share of public debt and to the output gap. We optimally choose the indicators that the fiscal and monetary authorities react to, as well as the magnitude of feedback policy reaction to those indicators. The welfare criterion is the household’s expected discounted lifetime utility. In this chapter we work within two environments. In the first, used as a benchmark, the authorities just stabilize the economy from shocks. In the second, the fiscal authorities also aim at gradually reducing output share of public debt over time, which means that now we combine shock stabilization with resource allocation policy. The second case/environment studied is related to the literature on debt consolidation.

Chapter 4 builds upon the closed model of the previous chapter. Particularly, the setup is a New Keynesian model of a semi-small open economy with sovereign risk premia. These sovereign premia mean that the interest rate, at which the country borrows from the world capital market, increases with the government’s total debt. We focus on a monetary policy regime in which the small open economy fixes the exchange rate and, at the same time, loses monetary independence; this mimics membership in a currency union. Hence, the only macroeconomic tool left is fiscal policy. We then allow public spending and the main types of tax rates to respond to the gap
between actual public debt and target public debt as shares of output, as well as to the gap between actual and target output. We experiment with various targets depending on whether policymakers aim just to stabilize the economy around its status quo, or whether they also want to move the economy to a reformed long-run (for instance a new long run without sovereign premia). The model is solved numerically using fiscal and public finance data from the Italian economy during 2001-2011. To rank different policies, and since we do not want our results to be driven by ad hoc differences in feedback policy coefficients across different policy rules, we compute optimized policy rules when the welfare criterion is household’s expected lifetime utility.

Chapter 5 builds a New Keynesian DSGE model consisting of two heterogeneous countries participating in a monetary union. We study how public debt consolidation in a country with high debt and sovereign risk premia (like Italy) affects welfare in a country with solid public finances (like Germany) and how these effects depend on the fiscal policy mix chosen to bring public debt down.
2 Stabilizing monetary policy

2.1 Introduction

This chapter does three things. First, it presents a rather standard New Keynesian model featuring Calvo-type nominal fixities and imperfect competition in the product market. Second, it solves the model numerically by employing the computational methodology commonly used by the DSGE literature nowadays. Third, it reviews and then enriches the existing literature on the use of nominal interest rate (monetary) policy as a macroeconomic stabilization device.

Regarding the third aim of the paper, we compare various Taylor-type feedback interest-rate policy rules in order to evaluate the benefits and costs from reacting to a number of different macroeconomic indicators used to capture inflation and output performance. In this chapter, the criterion used to rank various policies is the volatility of output and inflation.

In particular, we focus on three key monetary policy questions. Firstly, we ask whether interest-rate policy should react, or not, to the output gap. We ask so because counter-cyclical reaction to the output gap may generate a harmful output inflation trade-off. Secondly, if, for some reason, monetary policy does react to the output gap, we examine which is the best operating target for output among several candidates suggested by the relevant literature, e.g. the steady-state level of output, the natural level of output, the efficient level of output, or the output growth rate. Thirdly, we ask what happens when monetary policy is implemented via an operational Taylor rule. An operational Taylor rule requires minimal information both for policymakers and the public in the sense that the nominal interest rate is set as a function of macroeconomic indicators which are easily observed and known at the time of policy action.

Our results are as follows. First, in the case in which the monetary authority uses the benchmark Taylor rule proposed by Taylor (1993), it should react only to deviations of actual inflation from its target, while it should not react to deviations of output from its target. This implies that the monetary authority should solely focus on undoing the distortion arising from nominal fixities. Second, if monetary policy for some reason reacts to the output gap, the "best" target is the natural level of output, i.e. the output that will prevail under flexible prices. However, the resulting gains are quantitatively small. Third, we find that an operational Taylor rule performs better than all Taylor-type rules we examine in this chapter.

The rest of this chapter is organized in five sections. Section 2 presents the model, the decentralized equilibrium and policy. Section 3 describes the calibration and the long-run solution. Section 4 presents the key results. Section 5 concludes. An Appendix includes details and additional results.

2.2 The Model

This section presents a rather standard New Keynesian model of a closed economy.

2.2.1 Households

There is a representative household indexed by $i$. Household $i$ gets utility from a consumption index, $c_{i,t}$, real money balances, $m_{i,t}$, and disutility from hours worked, $n_{i,t}$:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, m_{i,t}, n_{i,t})$$  \hspace{1cm} (1)

where $0 < \beta < 1$ is the time discount rate and $E_0$ is the rational expectations operator conditional on the current period information set.
We will use the following utility function:

\[ u_{i,t} = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} + \chi_m \frac{m_{i,t}^{1-\mu}}{1-\mu} - \chi_n \frac{n_{i,t}^{1+\eta}}{1+\eta} \] (2)

The period budget constraint of each household \( i \) in nominal terms is:

\[ P_t c_{i,t} + P_t x_{i,t} + B_{i,t} + M_{t} = R_{t-1} B_{i,t-1} + M_{i,t-1} + W_t n_t + P_t r_{k_{t-1}} - T_{i,t}^l + D_{i,t} \] (3)

where \( P_t \) is the general price index, \( B_{i,t-1} \) is a private non state-contingent nominal bond, \( R_{t-1} \) is the gross nominal interest rate, \( D_{i,t} \) are nominal lump-sum taxes, \( M_{i,t-1} \) are nominal money holdings, \( k_{i,t-1} \) is physical capital, \( W_t \) and \( P_t r_{k_{t}} \) are the nominal wage rate and the nominal return to capital respectively.

Dividing by \( P_t \), the budget constraint of each \( i \) in real terms is:

\[ c_{i,t} + x_{i,t} + b_{i,t} + m_{i,t} = R_{t-1} b_{i,t-1} + m_{i,t-1} - P_{t-1}^{\frac{1}{c}} + w_t n_t + r_{k_{t-1}} - r_{t}^{l} + d_{i,t} \] (4)

where small letters denote real variables \( b_{i,t} \equiv \frac{B_{i,t}}{P_t} \), \( m_{i,t} \equiv \frac{M_{i,t}}{P_t} \), \( w_t \equiv \frac{W_t}{P_t} \), \( r_{t}^{l} \equiv \frac{T_{i,t}^l}{P_t} \) and \( d_{i,t} \equiv \frac{D_{i,t}}{P_t} \).

Physical capital accumulates according to:

\[ k_{i,t} = (1-\delta) k_{i,t-1} + x_{i,t} \] (5)

where \( 0 < \delta < 1 \) is a depreciation rate.

Household \( i \)'s consumption bundle at \( t \), \( c_{i,t} \), is a composite of \( h = 1, 2, \ldots, N \) varieties of goods, denoted as \( c_{i,t}(h) \), where each variety \( h \) is produced monopolistically by one firm \( h \). Using a Dixit-Stiglitz aggregator, we define:

\[ c_{i,t} = \left[ \sum_{h=1}^{N} \lambda c_{i,t}(h) \right]^{\frac{1}{\sigma}} \] (6)

where \( \phi > 0 \) is the elasticity of substitution across goods produced and \( \sum_{h=1}^{N} \lambda = 1 \) are weights (to avoid scale effects, we assume \( \lambda = 1/N \)).

Household \( i \)'s total consumption expenditure is:

\[ P_t c_{i,t} = \sum_{h=1}^{N} \lambda P_t \alpha c_{i,t}(h) \] (7)

where \( P_t \alpha \) is the price of variety \( h \).

**Household’s optimality conditions** Each household \( i \) acts competitively taking prices and policy as given. Following the literature, to solve household’s problem, we follow a two-step procedure. Thus, we first suppose that the household chooses its desired consumption of the composite good \( c_{i,t} \), and, in turn chooses how to distribute its purchases of individual varieties, \( c_{i,t}(h) \). The first-order conditions include the constraints above and:

\[ \frac{c_{i,t}^{1-\sigma}}{(1+r_{t}^{l})} = \beta E_t \frac{c_{i,t+1}^{1-\sigma}}{(1+r_{t+1}^{l})} \left[ r_{k_{t+1}}^{l} + (1-\delta) \right] \] (8)
\[
\frac{c_{i,t}}{(1 + \tau_t)} = \beta E_t \frac{c_{i,t+1}}{(1 + \tau_{t+1})} R_t \frac{P_t}{P_{t+1}} 
\]

(9)

\[
\chi_n m_{i,t} = \frac{c_{i,t}}{(1 + \tau_t)} + \beta E_t \frac{c_{i,t+1}}{(1 + \tau_{t+1})} P_t = 0
\]

(10)

\[
\chi_n \frac{n_{i,t}}{c_{i,t}} = w_t
\]

(11)

\[
c_{i,t}(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} c_{i,t}
\]

(12)

and the transversality conditions.

Equations (8) and (9) are respectively the Euler equations for capital and private bonds respectively, (10) is the optimality condition for real money balances, (11) is the optimality condition for work hours and (12) shows the optimal demand for each variety of the composite good.

2.2.2 Firms

There are \( h = 1, 2, ..., N \) firms. Each firm \( h \) produces a differentiated good of variety \( h \) under monopolistic competition facing Calvo-type nominal fixities.

Demand for firm's product Each firm \( h \) faces demand for its product, \( y_t(h) \), coming from households' consumption and investment, \( c_t(h) \) and \( x_t(h) \), where \( c_t(h) \equiv \sum_{i=1}^{N} c_{i,t}(h) \) and \( x_t(h) \equiv \sum_{i=1}^{N} x_{i,t}(h) \), and from the government, \( g_t(h) \). Thus, the demand for each firm's product is:

\[
y_t(h) = c_t(h) + x_t(h) + g_t(h)
\]

(13)

where from above:

\[
c_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} c_t
\]

(14)

and similarly:

\[
x_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} x_t
\]

(15)

\[
g_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} g_t
\]

(16)

where \( c_t \equiv \sum_{i=1}^{N} c_{i,t}, x_t \equiv \sum_{i=1}^{N} x_{i,t} \) and \( g_t \) is public spending.

Since at the economy level,

\[
y_t = c_t + x_t + g_t
\]

(17)

the above equations imply that the demand for each firm's product is:

\[
y_t(h) = c_t(h) + x_t(h) + g_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} y_t
\]

(18)
Firm’s problem Each firm $h$ maximizes nominal profits, $D_t(h)$, defined as:

$$D_t(h) = P_t(h)y_t(h) - Pr_t^b k_{t-1}(h) - W_t n_t(h)$$  \hspace{1cm} (19)

All firms use the same technology represented by the production function:

$$y_t(h) = A_t[k_{t-1}(h)]^a[n_t(h)]^{1-a}$$ \hspace{1cm} (20)

where $A_t$ is an exogenous stochastic TFP process whose motion is defined below.

Under imperfect competition, profit maximization is subject to:

$$y_t(h) = rac{P_t(h)}{P_t} y_t$$ \hspace{1cm} (21)

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In each period, firm $h$ faces an exogenous probability $\theta$ of not being able to reset its price thus it sets its previous period price times steady-state inflation $\Pi P_{t-1}(h)$. A firm $h$, which is able to reset its price, chooses its price $P_t^b(h)$ to maximize the sum of discounted expected nominal profits for the next $k$ periods in which it may have to keep its price fixed.

Following the related literature, to solve the firm’s problem above, we follow a two-step procedure. We first solve a cost minimization problem, where each firm minimizes its cost by choosing factor inputs given technology and prices. The solution will give a minimum nominal cost function, which is a function of factor prices and output produced by the firm. In turn, given this cost function, each firm, which is able to reset its price, solves a maximization problem by choosing its price.

Cost minimization problem Firm $h$ solves the following cost minimization problem under the assumption that the factor input markets operate under perfect competition:

$$\Psi_t(y_t(h)) = \min \left\{ P_t w_t n_t(h) + Pr_t^b k_{t-1}(h) \right\}$$  \hspace{1cm} (22)

subject to:

$$y_t(h) = A_t[k_{t-1}(h)]^a[n_t(h)]^{1-a}$$ \hspace{1cm} (23)

In order to solve it we form the Lagrangian:

$$\mathcal{L}_t = P_t w_t n_t(h) + Pr_t^b k_{t-1}(h) - \psi_t(h) \left( A_t k_{t-1}(h)^a n_t(h)^{1-a} - y_t(h) \right)$$ \hspace{1cm} (24)

The first order conditions with respect to $n_t(h)$ and $k_{t-1}(h)$ respectively are:

$$P_t w_t - \psi_t(h)(1-a)A_t k_{t-1}(h)^a n_t(h)^{-a} = 0$$ \hspace{1cm} (25)

$$Pr_t^b - \psi_t(h)aA_t k_{t-1}(h)^a n_t(h)^{1-a} = 0$$ \hspace{1cm} (26)

Divide relation (25) with (26) we get:

\footnote{This is a common assumption in New Keynesian literature, see Ascari 2004.}
\[
\frac{w_t}{r^*_t} = \frac{(1-a)A_t k_{t-1}(h)^a n_t(h)^{-a}}{a A_t k_{t-1}(h)^{a-1} n_t(h)^{1-a}} = \frac{1-a}{a} \frac{k_{t-1}(h)}{n_t(h)}
\]

Thus, if we take the first derivative of the cost function, \(\Psi(.)\), with respect to \(y_t(h)\) we end up with the relation for nominal marginal cost is:

\[
\Psi'(y_t(h)) = (1-a)^{-1-a} (a)^{-a} (r^*_t)^a (w_t)^{1-a} \frac{P_t}{A_t}
\]

And if we plug (28) into (26) we can show that:
The solution to the cost minimization problem gives the input demand functions:

\[ w_t = mc_t (1 - a) A_t | k_{t-1} (h) |^a [n_t (h)]^{-\alpha} \]  

\[ r^k_t = mc_t a A_t | k_{t-1} (h) |^{a-1} [n_t (h)]^{-\alpha} \]  

where \( mc_t = \psi_t (h) \) is the real marginal cost with \( \Psi_t (.) \) denoting the associated minimum nominal cost function for producing \( y_t (h) \) at \( t \).

**Profit maximization** Now we turn to the profit maximization problem in which firms take as given the nominal profit function, \( \Psi_t (y_t (h)) \) given by (31). Then, the firm chooses its price, \( P_t^* (h) \), to maximize nominal profits written as:

\[
\max_{P_t} \sum_{k=0}^{\infty} (\theta)^k E_{t+k} D_{t+k} (h) = \sum_{k=0}^{\infty} (\theta)^k E_{t} \Xi_{t+k} \left\{ \Pi^k P_t^* (h) y_{t+k} (h) - \Psi_{t+k} (y_{t+k} (h)) \right\}
\]

where \( \Xi_{t+k} \) is a discount factor taken as given by the firm and where \( y_{t+k} (h) = \left[ \Pi^k P_t^* (h) P_{t+k} \right]^{-\phi} \).

The first-order condition gives:

\[
\sum_{k=0}^{\infty} (\theta)^k E_{t} \Xi_{t+k} \left\{ \Pi^k P_t^* (h) \right\}^{-\phi} y_{t+k} \left\{ \Pi^k P_t^* (h) - \phi - 1 \Psi_{t+k} \right\} = 0 \]  

(36)

We transform the above equation by dividing by the aggregate price index, \( P_t \):

\[
\sum_{k=0}^{\infty} (\theta)^k E_{t} [\Xi_{t+k} \left\{ \Pi^k P_t^* (h) \right\}^{-\phi} y_{t+k} \left\{ \Pi^k P_t^* (h) - \phi - 1 mc_t \right\} P_{t+k} \]  

(37)

Therefore, the behaviour of each firm \( h \) is summarized by the above three conditions (34), (35) and (37). Each firm \( h \) which can reset its price in period \( t \) solves an identical problem, so \( P_t^* (h) = P_t^* \) is independent of \( h \), and each firm \( h \) which cannot reset its price just sets its previous period price times steady-state inflation, \( P_t (h) = \Pi P_{t-1} (h) \). Then, it can be shown (see Gali 2008) that the evolution of the aggregate price level is given by:

\[
(P_t)^{1-\phi} = \theta (\Pi P_{t-1})^{1-\phi} + (1 - \theta) (P_t^*)^{1-\phi} \]  

(38)

**2.2.3 Government**

The government solves an identical problem to household \( i \) so that it allocates its expenditure among differentiated goods \( h \):

\[
g_t = \left[ \sum_{h=1}^{N} \lambda (g_t (h)) \right]^{\frac{1}{1+\phi}} \]  

(39)

Thus, demand for good \( h \) from government is given by:

\[
g_t (h) = \left[ \frac{P_t (h)}{P_t} \right]^{-\phi} g_t \]  

(40)
The government finances its expenditures by seignorage revenues and lump-sum taxes. Thus, the budget constraint in nominal terms is given by:

\[ T_t + M_t = M_{t-1} + P_t g_t \]  

(41)

where \( M_t \) is the end-of-period total stock of money balances, \( T_t \) is total lump-sum taxes and \( P_t g_t \) is nominal government spending. We also have \( M_t \equiv \sum_{i=1}^{N} M_{i,t} \) and \( T_t \equiv \sum_{i=1}^{N} T_{i,t} \).

2.2.4 Decentralized Equilibrium (given policy)

We now present the Decentralized Equilibrium (DE) for any feasible monetary and fiscal policy. This DE is summarized by the following equilibrium conditions:

\[ \frac{c_t^{-\sigma}}{(1 + \tau_t^{-\sigma})} = \beta E_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^{-\sigma})} \left( r_{t+1}^k + (1 - \delta) \right) \]

(42)

\[ c_t^{-\sigma} = \beta E_t R_t c_t^{-\sigma} \frac{P_t}{P_{t+1}} \]

(43)

\[ \chi_m c_t^{-\sigma} - c_t^{-\sigma} + \beta E_t c_t^{-\sigma} \frac{P_t}{P_{t+1}} = 0 \]

(44)

\[ \chi_n \frac{n_t}{c_t^{-\sigma}} = w_t \]

(45)

\[ k_t = (1 - \delta) k_{t-1} + x_t \]

(46)

\[ \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Xi_{t,t+k} \left( \Pi^k \frac{P_t}{P_{t+k}} \right)^{-\phi} y_{t+k} \left( \Pi^k \frac{P_t}{P_{t+k}} - \frac{\phi}{\phi - 1} m_{t+k} \frac{P_{t+k}}{P_t} \right) \right\} = 0 \]

(47)

\[ w_t = mc_t(1 - a) \frac{y_t}{n_t} \]

(48)

\[ r_t^k = mc_t a \frac{y_t}{k_t} \]

(49)

\[ d_t = y_t - w_t n_t - r_t^k k_{t-1} \]

(50)

\[ y_t = \frac{1}{\left( \frac{P_t}{P_{t+1}} \right)^{-\phi}} A_t k_{t+1}^a n_t^{1-a} \]

(51)

\[ m_t = m_{t-1} \frac{P_{t-1}}{P_t} g_t - \tau_t^l \]

(52)

\[ y_t = c_t + x_t + g_t \]

(53)

\[ (P_t)^{1-\phi} = \theta (P_{t-1})^{1-\phi} + (1 - \theta) \left( P_t^# \right)^{1-\phi} \]

(54)

\[ (\tilde{P}_t)^{-\phi} = \theta (\Pi \tilde{P}_{t-1})^{-\phi} + (1 - \theta) \left( P_t^# \right)^{-\phi} \]

(55)

where \( \tilde{P}_t \equiv \left( \sum_{h=1}^{N} [P_t (h)]^{-\phi} \right)^{-\frac{1}{\phi}} \) and \( \left( \frac{P_t}{P_{t+1}} \right)^{-\phi} \) is a measure of price dispersion.
We thus have a system of 14 equilibrium conditions. To solve this system, we need to specify
the policy regime and thus classify policy instruments into endogenous and exogenous. Regarding
the conduct of monetary policy, we assume that the nominal interest rate, $R_t$, is used as a
policy instrument, while, regarding fiscal policy, we assume that the residually determined public
financing policy instrument is lump-sum taxes, $l_t$. In this case, the 14 endogenous variables are
\{y_t, c_t, x_t, k_t, m_t, b_t, P_t, P^#_t, \tilde{P}_t, w_t, mc_t, d_t, r^g_t\}_{t=0}^\infty. This is given the independently
set policy instruments, \{R_t, g_t\}_{t=0}^\infty, aggregate technology, \{A_t\}_{t=0}^\infty, and initial conditions for
the state variables. Notice that the DE system is an extension of the standard textbook New
Keynesian model, e.g. Gali (2008), which can be reduced after log-linearizing it to a linear system
of three equations in three unknowns. In our case, this cannot be done because of the presence
of capital and real money balances.

2.2.5 The conduct of monetary and fiscal policy and their role

We assume that monetary policy follows a simple Taylor-type feedback rule. Starting with the
baseline Taylor rule proposed by Taylor (1993), we have:
$$
\log \frac{R_t}{R} = \phi_x \log \Pi_t + \phi_y \log \frac{y_t}{y}
$$
where variables without time subscript denote steady-state values. Below, we experiment with
alternative Taylor-type rules.

To focus on monetary policy issues, we keep fiscal policy simple. Thus, the fiscal authority
sets the government spending output ratio ($s^g t \equiv g_t / y_t$) equal to its steady-state value for all periods:
$$
s^g t = s^g
$$

In this model, there are two market distortions; market power in goods market and infre-
quently adjusting prices. The first distortion is in the goods market, firms perceive the demand
for its differentiated product to be imperfectly elastic, this endows them with some market power
so the price of the produced good is set with a markup above its marginal cost. The presence
of a markup distortion leads to an inefficiently low level of output. The second distortion is the
Calvo-type staggered price setting which creates two sources of inefficiency. First, firms are not
allowed to adjust their prices continuously, which causes the economy’s average markup to vary
over time and differ from the constant markup that will prevail under flexible prices. This source
of inefficiency implies either too low or too high level of aggregate output. Second, the presence
of Calvo-type nominal fixity implies that the relative prices of different goods can vary in a way
unrelated to changes in preferences or/and technology. Such relative price distortions will lead
to different quantities of the different goods being produced and consumed and this is inefficient.
In this chapter, we focus on the stabilizing properties of monetary policy.\(^2\).

2.2.6 Exogenous shocks

We assume that the economy fluctuates around this steady-state due to TFP shocks which follow
an AR(1) process:
$$
\log (A_t) = (1 - \rho^a) \log (A) + \rho^a \log (A_{t-1}) + \epsilon^a_t
$$
\(^2\)In the textbook New Keynesian model the inefficiency resulting from the presence of market power is usually
eliminated through the suitable choice of an employment subsidy. Given this subsidy, the efficient allocation can
be attained by a monetary policy that stabilizes marginal costs at a level consistent with firms’ desired markup.
where \( 0 < \rho^a < 1 \) is a parameter, variables without time subscript denote long-run values and 
\( \varepsilon^a_t \sim N(0, \sigma^a) \).

Now, we solve for the long-run of the model by using conventional values and data from eurozone. Then, we take a first order approximation around this long-run and we study transitional dynamics. Particularly, we use standard DGSE routines to obtain the policy functions (see Uhlig (1997), Klein (2000), Schmitt-Grohè and Uribe (2004) and Dynare).

2.3 Data, parameterization and the long-run solution

This subsection solves numerically for the long-run of the above economy using conventional parameter values and data from the euro zone.

2.3.1 Data and parameterization

This section parameterizes the model to the Euro area over the period 2001-2010. The structural parameter values are summarized in Table 1.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Preference</td>
<td>( \beta )</td>
<td>0.9926</td>
</tr>
<tr>
<td>Risk Aversion Coef.</td>
<td>( \sigma )</td>
<td>1</td>
</tr>
<tr>
<td>Depreciation Rate</td>
<td>( \delta )</td>
<td>0.021</td>
</tr>
<tr>
<td>Price Rigidity Parameter</td>
<td>( \theta )</td>
<td>3/4</td>
</tr>
<tr>
<td>Price Elasticity of Demand</td>
<td>( \varepsilon )</td>
<td>6</td>
</tr>
<tr>
<td>Share of Capital</td>
<td>( \alpha )</td>
<td>1/3</td>
</tr>
<tr>
<td>Elasticity of Real Money Balances</td>
<td>( \nu )</td>
<td>3.42</td>
</tr>
<tr>
<td>Labour Elasticity</td>
<td>( \phi )</td>
<td>1</td>
</tr>
<tr>
<td>Government spending as a share of output</td>
<td>( s^g )</td>
<td>0.2</td>
</tr>
<tr>
<td>Persistence of TFP</td>
<td>( \rho^a )</td>
<td>0.8</td>
</tr>
<tr>
<td>Standard deviation of TFP</td>
<td>( \sigma^a )</td>
<td>0.0062</td>
</tr>
</tbody>
</table>

The value of the time preference rate, \( \beta \), follows from setting \( R = 1.01237 \) and \( \Pi = 1.0^{20.25} \).

We assume that the government spending to output ratio is \( s^g = 0.2 \), which is a value close to the euro-zone data average 2001-2011. The elasticity of intertemporal substitution, \( \sigma \), the depreciation rate of capital, \( \delta \), the price elasticity of demand, \( \varepsilon \), the share of capital, \( \alpha \), and the inverse of Frisch labour elasticity, \( \phi \), are all taken from Andrès and Domenech (2006). The real money balances elasticity, \( \nu \), is taken from Pappa and Neiss (2005). The price rigidity parameter, \( \theta \), is taken from Gali (2008). Concerning the exogenous stochastic variables, we set \( \rho^a = 0.8 \) and \( \sigma^a = 0.0062 \) for the persistence and standard deviation respectively of the TFP shock which are as in Andrès and Domenech (2006).

2.3.2 Long-run solution

Table 2 reports the long-run solution of the model economy when we use the parameter values and the policy instruments in Table 1\(^3\). The solution is well defined and makes sense, since the resulting great ratios are close to their values in the data. The time unit is meant to be a quarter, thus stock variables are divided by 4 to give annual values.

\(^3\)Details of the solution are in Appendix B.
Table 2: Long-run solution

<table>
<thead>
<tr>
<th>Endogenous variables</th>
<th>Steady-state solution</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3.019</td>
<td>-</td>
</tr>
<tr>
<td>c</td>
<td>1.796</td>
<td>-</td>
</tr>
<tr>
<td>x</td>
<td>1.005</td>
<td>1.005</td>
</tr>
<tr>
<td>( \frac{z}{y} )</td>
<td>0.59</td>
<td>0.57</td>
</tr>
<tr>
<td>( \frac{m}{y} )</td>
<td>1.42</td>
<td>1.6</td>
</tr>
<tr>
<td>( \frac{\theta}{y} )</td>
<td>0.2</td>
<td>0.18</td>
</tr>
</tbody>
</table>

2.4 How we work

In the next subsections we evaluate Taylor-type rules. To do so, we work in two steps. In the first step, we search for the ranges of feedback policy coefficients as defined in equation (56), which allow us to get a unique locally determinate equilibrium. In the second step, within the determinacy area found, we compare various Taylor-type feedback interest rate rules in order to evaluate the benefits and the costs from reacting to a number of different macroeconomic indicators. In this chapter, the criterion used to rank various policies is the volatility of output and inflation. To this end, we simulate the model 1000 times for 300 periods and we use them to compute simulated moments.

2.5 Results

This section presents numerical results. We start by presenting the determinacy areas. Then, we evaluate alternative Taylor-type rules.

2.5.1 Determinacy areas

We require the feedback policy coefficient in Taylor-rule (56) to guarantee a unique local equilibrium. Determinacy depends crucial on the values of the feedback policy coefficients. We report that economic policy guarantees determinacy only when the nominal interest rate reacts more than one to one to inflation, i.e. \( \phi_\pi > 1 \). On the other hand, feedback reaction on output, \( \phi_y \), is not necessary for determinacy but a positive reaction to the output gap enlarges determinacy area and allows monetary policy to decrease the feedback coefficient on output (see details in Appendix C, Figure 1).

2.5.2 React to the output gap or not?

We start with the Taylor rule (56), henceforth baseline Taylor rule. In this section we ask whether the monetary authority should react to output or not, i.e. \( \phi_y > 0 \). To this end, we compare the case in which the nominal interest rate does not react to the output gap, i.e. \( \phi_x = 3 \) and \( \phi_y = 0 \), with the case in which we switch on the countercyclical reaction to the baseline rule (56) by setting \( \phi_x = 3 \) and \( \phi_y = 0.5 \), which is a value consistent with empirical estimates for ECB (see Gerlach and Schnabel, 2000). We find that the standard deviation of output is reduced from 0.016 to 0.009 but inflation volatility is increased from 0.0005 to 0.0071. This implies that countercyclical reaction generates an inflation-output trade off. The monetary authority should react only to deviations of the actual inflation from its target so as to replicate the flexible price equilibrium, while it should not respond to the output gap because it generates inflation.
volatility. The intuition from this result is that monetary policy should focus on undoing the distortion related with nominal rigidity and avoid countercyclical reaction to the output gap (see also Schmitt-Grohé and Uribe 2007).

2.5.3 Alternative output targets

In this section, we experiment with different targets for output. In particular, the second term in equation (56) is now written as, \( \log \left( \frac{y_t}{y_T} \right) \), where \( y_T \) is a time-varying output target, which is compared to the baseline Taylor rule case in which the target of output was equal to its steady-state value all the time.

There are several targets, \( y_T \), proposed by the literature. In Table 3, we compare the standard deviation of inflation and output when \( y_T \) is respectively the natural level of output, which is the output that will prevail under flexible prices and imperfect competition (see first row); the efficient level of output, which is the output that will prevail under flexible prices and perfect competition (second row); last-period output so that the nominal interest rate reacts to the growth rate (third row); and the steady-state output which as the case studied above and is repeated here for the reader’s convenience (fourth row). In all cases, we keep the feedback policy coefficients on inflation, \( \phi_1 \), and output, \( \phi_2 \), constant and equal to 3 and 0.5 respectively.

The standard deviation of output and inflation is reported in the second column of Table 3. From Table 3, we can infer that the "best" target is the natural level of output, in the sense that it keeps the volatility of inflation at its lowest level (on the other hand, the resulting gains are not so quantitatively significant). Thus, in terms of the joint stabilization of inflation and output, it is better to react to the natural level of output, while the worst case is when the nominal interest rate reacts to steady-state output (see also Figures 3-4 in Appendix C). Finally, we report that the efficient level of output destabilizes inflation and leads to a burst of mean inflation (around 20%). This is an expected result, because the efficient level of output is always higher than the distorted level of output, so the nominal interest is always lower than its steady-state value and the mean inflation is very large.

<table>
<thead>
<tr>
<th>Output targets</th>
<th>Volatilities of output ((std(y_t))) and inflation ((std(\Pi_t)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Output</td>
<td>(std(y_t) = 0.0160)</td>
</tr>
<tr>
<td></td>
<td>(std(\Pi_t) = 0.0004)</td>
</tr>
<tr>
<td>Efficient Output</td>
<td>(std(y_t) = 0.0161)</td>
</tr>
<tr>
<td></td>
<td>(std(\Pi_t) = 0.0007)</td>
</tr>
<tr>
<td>Last Period Output</td>
<td>(std(y_t) = 0.0124)</td>
</tr>
<tr>
<td></td>
<td>(std(\Pi_t) = 0.0018)</td>
</tr>
<tr>
<td>Long-Run Output</td>
<td>(std(y_t) = 0.0093)</td>
</tr>
<tr>
<td></td>
<td>(std(\Pi_t) = 0.0071)</td>
</tr>
</tbody>
</table>

Figures 3-4 in Appendix C illustrate the surfaces of the volatility of inflation and output respectively, as functions of the two-dimensional feedback policy space, i.e. \((\phi_1, \phi_2)\). Each surface corresponds to a different output target. Particularly, the blue surface illustrates the case in which the output target is set equal to its steady-state value, the green surface illustrates the case in which the output target is set equal to its previous period value, and the brown surface illustrates the case in which the output target is set equal to the natural level of output.
2.5.4 Strict inflation targeting

The analysis in subsections (2.5.2) and (2.5.3) yields two important results concerning countercyclical monetary policy. First, counter-cyclical monetary policy increases significantly the volatility of inflation for all output targets proposed in the literature, except the natural output. Second, the correct choice of the target for output requires too much information from the part of policymakers, while it only achieves a small improvement in macroeconomic performance. In our setup, it is thus better not to respond to the output gap. The polar case of strict inflation targeting is interesting because it is simple and easy to communicate to the public. In order to achieve the strict inflation targeting case through the Taylor rule, we set $\phi_x \to \infty$ and $\phi_y = 0$. Thus, actual inflation is always equal to its target, while the standard deviation of output falls to 0.0157. Thus, strict inflation targeting not only achieves zero volatility of inflation, but it also leads to lower volatility in output than the baseline Taylor rule.

2.5.5 An operational Taylor rule

In this subsection, we examine the stabilization properties of an "operational" Taylor-type rule. Several papers, like Orphanides (2001), McCallum (1999) and McCallum and Nelson (1999), have disputed the ability of policy makers to observe the targets suggested by the theoretical models above, i.e. the natural or steady state levels of output. In addition, they have shown that policy institutions may estimate current inflation and output from real time data with statistically significant errors. Given this criticism, we now analyse the performance of an operational Taylor rule proposed by Schmitt-Grohé and Uribe (2007), in which the nominal interest rate is set simply as a function of previous periods growth rates, which are easily observable. Thus, we use the following rule:

$$\log \left( \frac{R_t}{R_{t-1}} \right) = \rho \log \left( \frac{R_{t-1}}{R_{t-2}} \right) + \phi_x \log \left( \frac{H_{t-1}}{H} \right) + \phi_y \left( \frac{y_{t-1}}{y_{t-2}} \right)$$

(59)

Such a Taylor rule (59) requires information that is already known in period $t$ with certainty. We simulate the new model when the policy reaction coefficients are chosen optimally as in Schmitt-Grohé and Uribe (2007), namely, $\rho = 0.77$, $\phi_x = 0.75$ and $\phi_y = 0.02$ (see Table 4, second column). Moreover, we relax optimality assumption and choose some standard values for the reaction coefficients which guarantee local stability, see third column. In Table 4, we observe that this rule (59) performs better than all the Taylor rules that we have examined so far and is very close to the case of strict inflation targeting (see Figures 5-6 in Appendix C).

<table>
<thead>
<tr>
<th>Volatilities</th>
<th>Optimal Operational</th>
<th>Standard Operational</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.0156</td>
<td>0.0151</td>
<td>0.0160</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.0001</td>
<td>0.0004</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

Notes: In the first column we set $\rho = 0.77, \phi_x = 0.75$ and $\phi_y = 0.02$ in the second column we set $\rho = 0, \phi_x = 1.5$ and $\phi_y = 0.1$, while the baseline Taylor rule is defined in (56) where the target of output is the natural output and we set $\phi_n = 1.5$ and $\phi_y = 0.5$

5For a Central Bank to respond to the natural output needs a structural model and estimation from real time data.

6Figures 5-6 illustrate the surfaces of the volatility of inflation and output respectively, as functions of the two-dimensional feedback policy space, i.e. $(\phi_x, \phi_y)$ when the monetary authority follows the operational Taylor rule given by (59).
2.5.6 Conclusions

We studied the stabilization properties of various monetary policy rules in a closed economy. The goal of monetary policy was the joint stabilization of inflation and output around their targets. Our main conclusions are: Firstly, monetary policy can guarantee the joint stabilization of inflation and output through the baseline Taylor rule with a strong enough reaction to inflation and no reaction to the output gap. Secondly, a countercyclical monetary stance generates a trade-off between output and inflation stabilization, except from the case in which the monetary authority reacts to deviations from the natural output (i.e. the output that will prevail under flexible prices). This result confirms the proposal that the monetary authority should focus on inflation targeting. Thirdly, the joint stabilization of inflation and output does not require the knowledge or estimation of unobservable variables. Given that the policy reaction coefficients can guarantee the Blanchard-Kahn condition, a good monetary policy can be implemented through simple and operational Taylor type rules meaning that the interest rate responds to easily observed targets already known at the time of monetary policy action. In the chapters that follow below, we will study the robustness of these policy results to richer economic environments, where there is also active fiscal policy. We will also derive the feedback policy coefficients optimally.
2.6 Appendix A

2.6.1 Decentralized Equilibrium

The decentralized equilibrium can be summarized by the following 14 equations which determine the behaviour of the following 14 endogenous variables \( \{y_t, c_t, n_t, m_t, x_t, k_t, mc_t, dt, w_t, r^k_t, P_t, P_t, P^1_t, \} \), the economy faces an exogenous productivity shock \( A_t \), and the variables \( R_t \) and \( s^g_t \) are the policy instruments:

\[
\frac{c_t^{-\sigma}}{(1 + \tau_t^e)} = \beta E_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^e)} \left( r^k_{t+1} + (1 - \delta) \right) \tag{60}
\]

\[
c_t^{-\sigma} = \beta E_t R_t c_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} \tag{61}
\]

\[
\chi_n m_t^{-\mu} - c_t^{-\sigma} + \beta E_t c_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} = 0 \tag{62}
\]

\[
\chi_n \frac{n_t}{c_t^{-\sigma}} = w_t \tag{63}
\]

\[
k_t = (1 - \delta) k_{t-1} + x_t \tag{64}
\]

\[
\sum_{k=0}^{\infty} (\beta \theta)^k E_t \left[ \Xi_{t+k} \left[ \Pi^k \frac{P^#_t}{P_{t+k}} \right]^{-\phi} y_{t+k} \left( \Pi^k \frac{P^#_t}{P_t} - \phi - 1 \right) mc_{t+k} \frac{P_{t+k}}{P_t} \right] = 0 \tag{65}
\]

\[
w_t = mc_t (1 - a) \frac{y_t}{n_t} \tag{66}
\]

\[
r_t^k = mc_t a \frac{y_t}{k_t} \tag{67}
\]

\[
d_t = y_t - w_t n_t - r_t^k k_{t-1} \tag{68}
\]

\[
y_t = \frac{1}{\left( \frac{P_t}{P_t^#} \right)^{-\phi} A_t k_{t-1}^a n_t^{1-a}} \tag{69}
\]

\[
m_t = m_{t-1} \frac{P_{t-1}}{P_t} + g_{t-1} r_t^1 \tag{70}
\]

\[
y_t = c_t + x_t + g_t \tag{71}
\]

\[
(P_t)^{1-\phi} = \theta (\Pi P_{t-1})^{1-\phi} + (1 - \theta) \left( P^#_t \right)^{1-\phi} \tag{72}
\]

\[
(\tilde{P}_t)^{-\phi} = \theta (\Pi \tilde{P}_{t-1})^{-\phi} + (1 - \theta) \left( P^#_t \right)^{-\phi} \tag{73}
\]
2.6.2 Transform the Decentralized Equilibrium

We rewrite the model by using inflation rates rather than price levels. Thus we define three new endogenous variables, which are the gross inflation rate $\Pi_t \equiv \frac{P_t}{P_{t-1}}$, the auxiliary variable $\Theta_t \equiv \frac{E_t}{P_t}$, and the price dispersion index $\Delta_t \equiv \left[ \frac{P^*}{P_t} \right]^{-\varepsilon}$. So (72) can be written as:

$$\Pi_t^{1-\varepsilon} = \theta + (1 - \theta) \left[ \frac{P_t^*}{P_{t-1}} \right]^{1-\varepsilon}$$ \hspace{1cm} (74)

$$\Pi_t^{1-\varepsilon} = \theta + (1 - \theta) [\Theta_t \Pi_t]^{1-\varepsilon}$$ \hspace{1cm} (75)

We obtain a relation for the evolution of the price dispersion if we divide (73) with $P_t^{-\varepsilon}$:

$$\left( \frac{P'_t}{P_t} \right)^{-\varepsilon} = \theta \left( \frac{P'_{t-1}}{P_{t-1}} \right)^{-\varepsilon} + (1 - \theta) \left( \frac{P'^*}{P_t} \right)^{-\varepsilon}$$

Then:

$$\left( \frac{P'_t}{P_t} \right)^{-\varepsilon} = \theta \left( \frac{P'_{t-1} P_{t-1}}{P_t} \right)^{-\varepsilon} + (1 - \theta) \left( \frac{P'^*}{P_t} \right)^{-\varepsilon}$$ \hspace{1cm} (76)

If we plug the definitions given above into (76) we have a relation for the evolution of the price dispersion:

$$\Delta_t = (1 - \theta) \Theta_t^{-\varepsilon} + \theta \Pi_t \Delta_{t-1}$$ \hspace{1cm} (77)

Finally relation (65) is written:

$$\sum_{k=0}^{\infty} (\beta \theta)^k E_t \left\{ \xi_{t, t+k} \left[ \frac{P_t^* P_t P_{t-1}}{P_t P_{t-1} P_{t-2}} \right]^{-\varepsilon} y_{t+k} \left( \frac{P_t P_{t-1}}{P_t P_{t-1}} - \frac{\phi}{\phi - 1} mc_{t+k} P_t \frac{P_t P_{t-2}}{P_{t-2} P_{t-1}} \right) \right\} = 0$$

$$\sum_{k=0}^{\infty} (\beta \theta)^k E_t \left\{ \xi_{t, t+k} \left[ \frac{\Theta_t}{\Pi_{t-1}} \right]^{-\varepsilon} y_{t+k} \left( \Theta_t \Pi_t - \frac{\phi}{\phi - 1} mc_{t+k} \Pi_{t+1} \ldots \Pi_{t+k} \right) \right\} = 0$$ \hspace{1cm} (78)

2.6.3 Transformed Decentralized Equilibrium

In equilibrium we have a vector of 13 endogenous variables (without profits),

$$\{ y_t, c_t, n_t, m_t, x_t, k_t, mc_t, d_t, w_t, r^t, \Pi_t, \Theta_t, \Delta_t, r^t \}$$

the economy faces an exogenous productivity shock $A_t$, and the variables $R_t$ and $s^n_t$ are the policy instruments:

$$\frac{c_t^{1-\sigma}}{(1 + \tau_t^e)} = \beta E_t \frac{c_{t+1}^{1-\sigma}}{(1 + \tau_{t+1}^e)} \left( r_{t+1}^e + (1 - \delta) \right)$$ \hspace{1cm} (79)
\[ c_t^\sigma = \beta E_t R_t c_{t+1}^\sigma \frac{P_t}{P_{t+1}} \quad (80) \]
\[ \chi_m m_t - c_t^\sigma + \beta E_t c_{t+1}^\sigma \frac{P_t}{P_{t+1}} = 0 \quad (81) \]
\[ \chi_n n_t = w_t \quad (82) \]
\[ k_t = (1 - \delta) k_{t-1} + x_t \quad (83) \]

\[ \sum_{k=0}^{\infty} (\beta \theta)^k E_t \left\{ \Xi_{t,t+k} \left[ \Theta_t \frac{1}{\prod_{i=1}^{k} \Pi_{t+i}} \right]^{-\varepsilon} y_{t+k} \left( \Theta_t \Pi_t - \frac{\phi}{\delta - 1} mc_{t+k} \prod_{i=0}^{k} \Pi_{t+i} \right) \right\} = 0 \quad (84) \]

\[ w_t = mc_t (1 - a) A_t k_{t-1}^{a} n_t^{-a} \quad (85) \]
\[ r_t^k = mc_t a A_t k_{t-1}^{a-1} n_t^{1-a} \quad (86) \]
\[ y_t = \alpha_t + x_t + s_t^g \quad (87) \]
\[ y_t = \frac{1}{\Delta_t} A_t k_{t-1}^{a} n_t^{1-a} \quad (88) \]
\[ k_t = (1 - \delta) k_{t-1} + x_t \quad (89) \]
\[ \Pi_t^{1-\varepsilon} = \theta \left( \Pi \right)^{1-\varepsilon} + (1 - \theta) (\Pi, \Theta_t)^{1-\varepsilon} \]
\[ \Delta_t = \theta \left( \Pi \right)^{-\varepsilon} \Delta_{t-1} \Pi_t^\varepsilon + (1 - \theta) \Theta_t \varepsilon \]
\[ \tau_t + m_t = \frac{m_{t-1}}{\Pi_t} + s_t^g y_t \quad (90) \]

Monetary policy sets the nominal interest rate \( R_t \) and fiscal policy \( s_t^g = \frac{\Delta_t}{\Pi_t} \).

### 2.7 Appendix B

In this appendix we solve for the steady state starting by writing the system above without time subscript.
2.7.1 DE in steady-state

\[
\frac{1}{R} = \beta \frac{1}{\Pi} \tag{91}
\]

\[
1 = \beta \left[ (1 - \delta) + r^k \right] \tag{92}
\]

\[
w = \chi_n \frac{n^\phi}{c^{-\sigma}} \tag{93}
\]

\[
\chi_m m^{-\mu} c^{-\sigma} + \beta c^{-\sigma} \frac{1}{\Pi} = 0 \tag{94}
\]

\[
\sum_{k=0}^{\infty} (\beta \theta)^k \left[ \frac{\Pi^k}{\Pi^r} \Theta \right]^{-\varepsilon} \left\{ \Pi^k \Theta - \frac{\varepsilon}{\varepsilon - 1} mc \Pi^k \right\} = 0 \tag{95}
\]

\[
w = mc(1 - a) \frac{y}{n} \tag{96}
\]

\[
r^k = mca \frac{y}{k} \tag{97}
\]

\[
y = c + x + s_y y \tag{98}
\]

\[
y = \frac{1}{\Delta} A k^a n^{1-a} \tag{99}
\]

\[
x = \delta k \tag{100}
\]

\[
\Pi^{1-\varepsilon} = \theta (II)^{1-\varepsilon} + (1 - \theta) (II \Theta)^{1-\varepsilon} \tag{101}
\]

\[
\Delta = \theta (II)^{-\varepsilon} \Delta II^{\varepsilon} + (1 - \theta) \Theta^{-\varepsilon} \tag{102}
\]

\[
\tau + m = \frac{m}{\Pi} + s_y y \tag{103}
\]

2.7.2 Solution for long run

We set the nominal interest rate in steady-state in order to obtain the long run inflation occurred in Eurozone the last decade given that the time preference parameter \(\beta\) is set as in Andrés and Domenech (2006). So the interest rate must satisfy equation (91):

\[
R = \frac{\Pi}{\beta} \tag{104}
\]

We use (92) to take a steady-state value for \(r^k\):

\[
r^k = \frac{1}{\beta} - (1 - \delta) \tag{105}
\]

Solving (101) for \(\Theta\) we take \(\Theta = 1\) and from (102) we get \(\Delta = 1\). Plugging these values in (95) we get the standard monopolistic competition result under a flexible price economy that the marginal cost in steady-state is equal to the inverse of the mark-up:

\[
mc = \frac{\varepsilon - 1}{\varepsilon} \tag{106}
\]
Finally, the remaining 9 endogenous variables \( \{y, c, n, x, w, k, m, s^l\} \) must satisfy the 9 equilibrium equations:

\[
\begin{align*}
    w &= \chi_n \frac{n^{\phi}}{c^{-\sigma}} \\
    \chi_m m^{-\mu} - c^{-\sigma} + \beta c^{-\sigma} \frac{1}{\Pi} &= 0 \\
    w &= mc(1 - a) \frac{y}{n} \\
    r^k &= mca \frac{y}{k} \\
    y &= c + x + s^g y \\
    y &= Ak^s n^{1-a} \\
    x &= \delta k \\
    s^l + m &= \frac{m}{\Pi} + s^g y
\end{align*}
\]

2.8 Appendix C

2.8.1 Determinacy

The determinacy for the baseline Taylor Rule is illustrated in Figure 1:

\[
\log \frac{R_t}{R} = \phi_x \log \frac{\Pi_t}{\Pi} + \phi_y \log \frac{y_t}{y}
\]

\footnote{We find convenient to denote \( s^l \equiv \frac{s}{y} \).}
The determinacy are for the baseline operational taylor rule is illustrated in Figure 2:

$$\log \left( \frac{R_t}{R_{t-1}} \right) = \rho \log \left( \frac{R_{t-1}}{R_{t-2}} \right) + \phi_1 \log \left( \frac{\Pi_{t-1}}{\Pi} \right) + \phi_y \left( \frac{y_{t-1}}{y_{t-2}} \right)$$  \hspace{1cm} (116)
2.8.2 Volatility

Figures 3-4 illustrate the surfaces of the volatility of inflation and output respectively, as functions of the two-dimensional feedback policy space, i.e. $(\phi_x, \phi_y)$. Each surface correspond to the case in which monetary policy uses a different output target. Particularly, the blue surface illustrates the case in which the output target is set equal to its steady-state value, the green surface illustrates the case in which the output target is set equal to its previous period value, and the brown surface illustrates the case in which the output target is set equal to the natural level of output. Figure 5-6 illustrate the surfaces of the volatility of inflation and output respectively, as functions of the two-dimensional feedback policy space, i.e. $(\phi_x, \phi_y)$ when the monetary authority follows the operational Taylor rule given by (116).
The feedback policy parameter space is $\phi_x, \phi_y \in [0, 3]$. 

Figure 3: Volatility of output under alternative output targets
The feedback policy parameter space is $\phi_\pi, \phi_\nu \in [0, 3]$. 
The feedback policy parameter space is $\phi_x \in [0, 2], \phi_y \in [0, 0.1]$
The feedback policy parameter space is $\phi_x \in [0, 2], \phi_y \in [0, 0.1]$
3 Optimized monetary and fiscal policy rules in a closed economy

3.1 Introduction

Policymakers use their instruments to react to economic conditions. For instance, it is usually assumed that central banks respond to inflation, the fiscal authorities to the state of public finances, and both of them to real economic activity. It is also believed that the use of fiscal policy is more complex than the use of monetary policy. As e.g. Leeper (2010) points out, one reason is that, while central banks have a single instrument at their disposal at least in normal times, namely, the nominal interest rate, governments can make use of many types of spending and tax policy instruments. But different fiscal policy instruments have different implications (see e.g. Coenen et al., 2012).

In this paper, we search for the best mix of monetary and fiscal policy actions when the policy role is twofold: to stabilize the economy against shocks and to improve resource allocation by gradually reducing the public debt burden over time. In order to do so, we welfare rank various fiscal policy instruments used jointly with interest rate policy.

In particular, we specify feedback rules for public spending as share of output, the tax rate on labor income, the tax rate on capital income and the tax rate on consumption that allow for a response to a number of macroeconomic variables used as indicators, when, at the same time, monetary policy can be used in a standard Taylor-type fashion. We optimally choose the indicators that the fiscal and monetary authorities should react to, as well as the magnitude of feedback policy reaction to those indicators. The welfare criterion is household’s expected lifetime utility. This type of policy is known as "optimized policy rules" (see e.g. Schmitt-Grohé and Uribe, 2005, 2007). We work within two environments. In the first, the economy is hit by supply and demand shocks, which means that we solve a pure stabilization policy problem. In the second, the fiscal authorities also aim at gradually reducing the output share of public debt over time, which means that now we combine shock stabilization with resource allocation policy.

The setup is a standard New Keynesian model of a closed economy featuring imperfect competition and Calvo-type nominal rigidities, which is extended to include a rather rich menu of state-contingent policy rules. The model is calibrated to match data from the euro area over 1995-2010. To solve the model and, in particular, to solve for welfare-maximizing policy, we adopt the methodology of Schmitt-Grohé and Uribe (2004), in the sense that we take a second-order approximation to both the equilibrium conditions and the welfare criterion. In turn, we compute the welfare-maximizing values of various feedback policy rules and the associated social welfare. This enables us to welfare rank alternative policies in a stochastic setup.

Our main results are as follows. First, concerning fiscal instruments, it is better to use public spending, rather than taxes, for shock stabilization and/or debt consolidation. In all cases studied, public spending scores the best in terms of expected lifetime utility, being followed by consumption taxes, then capital taxes and lastly labor taxes. Labor taxes are clearly the worst policy instrument to make use of.

Second, in all cases studied, the monetary authorities should react to price inflation and the fiscal authorities should react to public debt. In terms of magnitudes, the interest rate reaction to price inflation should be aggressive, namely, more than one-for-one as also implied by the Taylor principle, while the fiscal reaction to public debt should be mild in general (even in the case of debt consolidation), except from the case in which we use the capital tax rate. The latter happens because, in the very short run, the capital tax can work like a capital levy on existing wealth which is not so distorting relative to other taxes (see Chamley, 1986, and Judd, 1985, as well as the simulations of Altig et al., 2001, who have studied tax reforms in the US).
Third, monetary and fiscal policy reaction to the output gap is in general welfare-improving, other things equal. In other words, counter-cyclical fiscal policy is productive (this modifies the "consensus assignment" of e.g. Gordon and Leeper, 2005, and Kirsanova et al., 2009, and supports the arguments of Wren-Lewis, 2010, for the use of active fiscal policy in an economic downturn). This holds especially when extrinsic volatility is relatively high. It also holds even in the case of debt consolidation. The latter (namely, that policy reaction to the output gap is desirable even when the fiscal authorities want to bring the public debt ratio down) is explained by the fact that, since debt consolidation strategies may hurt the real economy, monetary and fiscal policy also need to be alert in real economic activity at the same time.

Fourth, except from the case in which we use a particularly distorting policy instrument like the labor tax rate, debt consolidation is welfare superior to non debt consolidation, other things equal. This is despite the fact that debt consolidation comes at the cost of lower public spending, or higher taxes, during the early phase of the transition period. Also, the duration of the debt consolidation period, and so how quickly the debt should be brought down, depends on which fiscal instrument we use. The more distorting is the instrument used, the longer the period should be. For instance, if we use the public spending ratio to reduce the debt ratio from 85% to 60%, this should be within 40 quarters. At the other end, if we use the labor tax rate, it should take more than 100 quarters.

Fifth, since, in most cases, it is optimal for policy instruments to respond to several indicators at the same time, the central issue is which response should be the dominant one. It is the latter that will shape the net change in a particular policy instrument. Say, for instance, that the economy is hit by an adverse TFP shock causing at impact an economic downturn and a rise in the inherited public debt to output ratio. Then, in normal times during which macro volatility is relatively low and shock stabilization is the only policy goal, our impulse response functions show that, at impact, public spending should fall, and capital taxes should rise, to address the rise in the debt ratio. By contrast, consumption and labor taxes should be reduced at impact to address the fall in output. In other words, when, for some political economy reason, we have to use a relatively distorting policy instrument, like consumption and especially labor taxes, net policy changes should be dominated by the concern for output cycles and only over the medium term, when the adverse shock fades away, these policy instruments should be used to address debt cycles. These results become stronger when macro volatility is relatively high. Actually, now, all fiscal policy instruments, including public spending and capital taxes, should give priority to the output cycle over the short term. Nevertheless, these results are reversed when public debt consolidation is added to the policy goals. Now, irrespectively of the degree of macro volatility, all fiscal policy instruments should be earmarked for bringing the public debt ratio down, even during the early phase of economic downturn, and, as said above, this is welfare superior other things equal, except if we have to use a particularly distorting policy instrument like labor taxes.

How does our work differ? Although there has been a rich literature on the interaction between fiscal and monetary policy, there has not been a welfare comparison of all main tax-spending policy instruments in a unified framework, and how this comparison depends on the degree of extrinsic volatility and/or the policy goals.\(^8\)

---


\(^9\)Papers related to ours include Schmitt-Grohé and Uribe (2007), Batini et al. (2008), Bi (2010), Bi and Kumhof (2011), Herz and Hohberger (2012) and Cantore et al. (2012). Some details are as follows. Schmitt-Grohé and Uribe (2007) allow total tax revenues to respond to public debt. But they do not welfare rank different fiscal policy instruments. The same applies to Batini et al. (2008) who allow tax revenues as share of output to react to public debt and output. Bi (2010) does welfare rank different tax policy instruments. But she works in a real small open economy without monetary policy. Also, she allows the tax rates to respond to public debt.
The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents the data, calibration and the long-run solution. Section 4 explains how we work. Section 5 studies the case with shock stabilization only. Section 6 studies the case with shock stabilization and debt consolidation. Section 7 closes the paper.

3.2 Model

The model is a conventional New Keynesian model featuring imperfect competition and Calvo-type nominal rigidities, which is extended to include a rather rich menu of state-contingent policy rules.

3.2.1 Households

There are \( i = 1, 2, \ldots, N \) households. Each household \( i \) acts competitively to maximize expected lifetime utility.

Household’s problem  Household \( i \)’s expected lifetime utility is:

\[
E_0 \sum_{t=0}^{\infty} \beta^t U \left( c_{i,t}, n_{i,t}, m_{i,t}, g_t \right)
\]

where \( c_{i,t} \) is \( i \)'s consumption bundle (defined below), \( n_{i,t} \) is \( i \)'s hours of work, \( m_{i,t} = \frac{M_{i,t}}{P_t} \) is \( i \)'s real money balances, \( g_t \) is per capita public spending, \( 0 < \beta < 1 \) is the time discount rate, and \( E_0 \) is the rational expectations operator conditional on the current period information set. In our numerical solutions, we use the period utility function (see also e.g. Gali, 2008):

\[
U_{i,t} \left( c_{i,t}, n_{i,t}, m_{i,t}, g_t \right) = c_{i,t}^{1-\sigma} - \frac{n_{i,t}^{1+\eta}}{1+\eta} + \frac{m_{i,t}^{1-\mu}}{1-\mu} + \frac{g_t^{1-\zeta}}{1-\zeta}
\]

where \( \chi_\eta, \chi_m, \chi_g, \sigma, \eta, \mu, \zeta \) are preference parameters.

The period budget constraint of each household \( i \) is in nominal terms:

\[
(1 + \tau^c_i) P_t c_{i,t} + P_t x_{i,t} + B_{i,t} + M_{i,t} =
\]

\[
(1 - \tau^c_i) (r^b_i P_t k_{i,t-1} + D_{i,t}) + (1 - \tau^d_i) W_t n_{i,t} + R_{t-1} B_{i,t-1} + M_{i,t-1} - T^d_{i,t}
\]

where \( P_t \) is the general price index, \( x_{i,t} \) is \( i \)'s real investment, \( B_{i,t} \) is \( i \)'s end-of-period nominal government bonds, \( M_{i,t} \) is \( i \)'s end-of-period nominal money holdings, \( \tau^c_i \) is the real return to inherited capital, \( k_{i,t-1} \), \( D_{i,t} \) is \( i \)'s nominal dividends paid by firms, \( W_t \) is the nominal wage rate, \( R_{t-1} \) is the gross nominal return to government bonds between \( t-1 \) and \( t \), \( T^d_{i,t} \) is nominal lump-sum taxes/transfers to each \( i \) from the government, and \( \tau^c_i, \tau^b_i, \tau^d_i \) are respectively tax rates on private consumption, capital income and labour income.

only. Bi and Kumhof (2011) focus on the importance of liquidity-constrained households. Also, they do not rank different tax-spending feedback policy rules. Herz and Hohberger (2012) include monetary policy but, concerning fiscal policy, they only use public spending for stabilization. They also work in a linear-quadratic setup with an ad hoc policy objective function. Cantore et al. (2012) have a rich analysis studying optimal policy in abnormal times, but they assume that all tax policy instruments change by the same proportion. They also work with a linear-quadratic approximation.

10 For the New Keynesian model, see the textbooks of Gali (2008) and Wickens (2008).
Dividing by $P_t$, the budget constraint of each $i$ in real terms is:

\[
(1 + \tau_t^i) c_{i,t} + x_{i,t} + b_{i,t} + m_{i,t} = (1 - \tau_t^i) \left( r_t^i k_{i,t-1} + d_{i,t} \right) + (1 - \tau_t^i) w_t m_{i,t-1} + R_{t-1} P_t^{-1} b_{i,t-1} + P_{t-1}^{-1} m_{i,t-1} - \tau_t^i
\]

(120)

where small letters denote real variables, i.e. $m_{i,t} \equiv M_{i,t} / P_t$, $b_{i,t} \equiv B_{i,t} / P_t$, $w_t \equiv W_t / P_t$, $d_{i,t} \equiv D_{i,t} / P_t$, $l_{i,t} \equiv T_{i,t} / P_t$, at individual level.

The motion of physical capital for each household $i$ is:

\[
k_{i,t} = (1 - \delta) k_{i,t-1} + x_{i,t}
\]

(121)

where $0 < \delta < 1$ is the depreciation rate of capital.

Household $i$'s consumption bundle at $t$, $c_{i,t}$, is a composite of $h = 1, 2, ..., N$ varieties of goods, denoted as $c_{i,t}(h)$, where each variety $h$ is produced monopolistically by one firm $h$. Using a Dixit-Stiglitz aggregator, we define:

\[
c_{i,t} = \left[ \sum_{h=1}^{N} \lambda_c c_{i,t}(h) \right]^{\frac{1}{1 - \phi}}
\]

(122)

where $\phi > 0$ is the elasticity of substitution across goods produced and $\sum_{h=1}^{N} \lambda = 1$ are weights (to avoid scale effects, we assume $\lambda = 1/N$).

Household $i$'s total consumption expenditure is:

\[
P_t c_{i,t} = \sum_{h=1}^{N} P_t(h) c_{i,t}(h)
\]

(123)

where $P_t(h)$ is the price of variety $h$.

**Household’s optimality conditions** Each household $i$ acts competitively taking prices and policy as given. Following the literature, to solve the household’s problem, we follow a two-step procedure. Thus, we first suppose that the household chooses its desired consumption of the composite good, $c_{i,t}$, and, in turn, chooses how to distribute its purchases of individual varieties, $c_{i,t}(h)$. Details are available upon request. Then, the first-order conditions include the budget constraint above and:

\[
\frac{c_{i,t}^{-\sigma}}{(1 + \tau_t^i)} = \beta E_t \frac{c_{i,t+1}^{-\sigma}}{(1 + \tau_{t+1}^i)} \left[ (1 - \tau_t^k) r_t^k + (1 - \delta) \right]
\]

(124)

\[
\frac{c_{i,t}^{-\sigma}}{(1 + \tau_t^i)} = \beta E_t \frac{c_{i,t+1}^{-\sigma}}{(1 + \tau_{t+1}^i)} R_t \frac{P_t}{P_{t+1}}
\]

(125)

\[
x_t m_{i,t}^{-\mu} = \frac{c_{i,t}^{-\sigma}}{(1 + \tau_t^i)} + \beta E_t \frac{c_{i,t+1}^{-\sigma}}{(1 + \tau_{t+1}^i)} \frac{P_t}{P_{t+1}} = 0
\]

(126)

\[
x_t \frac{c_{i,t}}{c_{i,t}} = \frac{(1 - \tau_t^n)}{(1 + \tau_t^i)} w_t
\]

(127)

\[
c_{i,t}(h) = \left[ \frac{P_t(h)}{P_t} \right]^{\phi} c_{i,t}
\]

(128)
Equations (124) and (125) are respectively the Euler equations for capital and bonds, (126) is the optimality condition for money balances, (127) is the optimality condition for work hours and (128) shows the optimal demand for each variety of goods.

3.2.2 Implications for price bundles

Equations (123) and (128) imply that the general price index is (see also e.g. Wickens, 2008, chapter 7):

\[ P_t = \left[ \sum_{h=1}^{N} \lambda [P_t(h)]^{1-\phi} \right]^{\frac{1}{1-\phi}} \]  

(129)

3.2.3 Firms

There are \( h = 1, 2, \ldots, N \) firms. Each firm \( h \) produces a differentiated good of variety \( h \) under monopolistic competition facing Calvo-type nominal fixities.

**Demand for firm’s product** Each firm \( h \) faces demand for its product, \( y_t(h) \), coming from households’ consumption and investment, \( c_t(h) \) and \( x_t(h) \), where \( c_t(h) \equiv \sum_{i=1}^{N} c_{t,i}(h) \) and \( x_t(h) \equiv \sum_{i=1}^{N} x_{t,i}(h) \), and from the government, \( g_t(h) \). Thus, the demand for each firm’s product is:

\[ y_t(h) = c_t(h) + x_t(h) + g_t(h) \]  

(130)

where from above:

\[ c_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} c_t \]  

(131)

and similarly:

\[ x_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} x_t \]  

(132)

\[ g_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} g_t \]  

(133)

where \( c_t \equiv \sum_{i=1}^{N} c_{t,i} \), \( x_t \equiv \sum_{i=1}^{N} x_{t,i} \) and \( g_t \) is public spending.

Since, at the economy level:

\[ y_t = c_t + x_t + g_t \]  

(134)

the above equations imply that the demand for each firm’s product is:

\[ y_t(h) = c_t(h) + x_t(h) + g_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} y_t \]  

(135)
**Firm’s problem**  Each firm \( h \) maximizes nominal profits, \( D_t(h) \), defined as:

\[
D_t(h) = P_t(h)y_t(h) - P_t r_t^k k^2_{t-1}(h) - W t n_t(h)
\]  

(136)

All firms use the same technology represented by the production function:

\[
y_t(h) = A_t[k^1_{t-1}(h)]^\alpha [n_t(h)]^{1-\alpha}
\]  

(137)

where \( A_t \) is an exogenous stochastic TFP process whose motion is defined below.

Under imperfect competition, profit maximization is subject to:

\[
y_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} y_t
\]

(138)

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In each period, firm \( h \) faces an exogenous probability \( \theta \) of not being able to reset its price. A firm \( h \), which is able to reset its price, chooses its price \( P^*_t(h) \) to maximize the sum of discounted expected nominal profits for the next \( k \) periods in which it may have to keep its price fixed\(^{11}\).

**Firm’s optimality conditions**  Following the related literature, to solve the firm’s problem above, we follow a two-step procedure. We first solve a cost minimization problem, where each firm \( h \) minimizes its cost by choosing factor inputs given technology and prices. The solution will give a minimum nominal cost function, which is a function of factor prices and output produced by the firm. In turn, given this cost function, each firm, which is able to reset its price, solves a maximization problem by choosing its price.

The solution to the cost minimization problem gives the input demand functions:

\[
w_t = mc_t (1-\alpha)A_t k^1_{t-1}(h) [n_t(h)]^{-\alpha}
\]

(139)

\[
r^k_t = mc_t A_t [k^1_{t-1}(h)]^{\alpha - 1} [n_t(h)]^{1-\alpha}
\]

(140)

where \( mc_t = \Psi_t(\cdot) \) is the marginal nominal cost with \( \Psi_t(\cdot) \) denoting the associated minimum nominal cost function for producing \( y_t(h) \) at \( t \).

Then, the firm chooses its price, \( P^*_t(h) \), to maximize nominal profits written as:

\[
\max \sum_{k=0}^\infty (\theta)^k E_t \Xi_{t,t+k} D_{t+k}(h) = \sum_{k=0}^\infty (\theta)^k E_t \Xi_{t,t+k} \left\{ P^*_t(h) y_{t+k}(h) - \Psi_{t+k}(y_{t+k}(h)) \right\}
\]

where \( \Xi_{t,t+k} \) is a discount factor taken as given by the firm and where \( y_{t+k}(h) = \left[ \frac{P^*_t(h)}{P_{t+k}} \right]^{-\phi} y_{t+k} \).

The first-order condition gives:

\[
\sum_{k=0}^\infty (\theta)^k E_t \Xi_{t,t+k} \left[ \frac{P^*_t(h)}{P_{t+k}} \right]^{-\phi} y_{t+k} \left\{ P^*_t(h) - \frac{\phi}{\phi-1} \Psi_{t+k} \right\} = 0
\]

(141)

We transform the above equation by dividing with the aggregate price index, \( P_t \):

\[
\sum_{k=0}^\infty (\theta)^k E_t \Xi_{t,t+k} \left[ \frac{P^*_t(h)}{P_{t+k}} \right]^{-\phi} y_{t+k} \left\{ \frac{P^*_t(h)}{P_t} - \frac{\phi}{\phi-1} mc_{t+k} \frac{P_{t+k}}{P_t} \right\} = 0
\]

(142)

\(^{11}\)In Appendix B we solve the profit maximization problem when firm \( h \) faces Rotemberg-type nominal fixity.
Therefore, the behaviour of each firm \( h \) is summarized by the above three conditions (??), (140) and (142).

Each firm \( h \) which can reset its price in period \( t \) solves an identical problem, so \( P^h_t(h) = P^g_t \) is independent of \( h \), and each firm \( h \) which cannot reset its price just sets its previous period price \( P_t(h) = P_{t-1}(h) \). Then, it can be shown that the evolution of the aggregate price level is given by:

\[
(P_t)^{1-\phi} = \theta(P_{t-1})^{1-\phi} + (1-\theta) \left( P^g_t \right)^{1-\phi}
\]  

(143)

### 3.2.4 Government budget constraint

Government’s within-period budget constraint is (in aggregate nominal terms):

\[
B_t + M_t = R_{t-1}B_{t-1} + M_{t-1} + P_tg_t - \tau^h_tP_t\epsilon_t - \tau^k_t(r^h_tP_tk_{t-1} + D_t) - \tau^w_tW_tn_t - T^i_t
\]

(144)

where \( B_t \) is the end-of-period total domestic nominal public debt, \( M_t \) is the end-of-period total stock of money balances. We also have \( c_t \equiv \sum_{i=1}^N c_{it}, k_{t-1} \equiv \sum_{i=1}^N k_{it-1}, D_t \equiv \sum_{i=1}^N D_{it}, n_t \equiv \sum_{i=1}^N n_{it}, B_{t-1} \equiv \sum_{i=1}^N B_{it-1} \) and \( T^i_t \equiv \sum_{i=1}^N T^i_{it} \), and all other variables have been defined above. Also recall that the government allocates its total expenditure among product varieties \( h \) by solving an identical problem with household \( i \), so that \( g_t(h) = \frac{P_t(h)}{P_t} \). In each period, one of the fiscal policy instruments \((\tau^h_t, \tau^k_t, \tau^w_t, g_t, T^i_t, B_t)\) has to follow residually to satisfy the government budget constraint.

Dividing by \( P_t \), the government budget constraint is rewritten in real terms as:

\[
b_t + m_t = R_{t-1}P^{-1}_t\frac{P_t}{P_t}b_{t-1} + \frac{P_t}{P_t}m_{t-1} + g_t - \tau^h_t\epsilon_t - \tau^k_t(r^h_tk_{t-1} + d_t) - \tau^w_tw_tn_t - \tau^i_t
\]

(145)

where \( b_t \equiv \frac{b_t}{P_t}, m_t \equiv \frac{M_t}{P_t}, d_t \equiv \frac{D_t}{P_t}, w_t \equiv \frac{W_t}{P_t} \) and \( \tau^i_t \equiv \frac{T^i_t}{P_t} \).

### 3.2.5 Decentralized equilibrium (for any feasible policy)

We now combine the above to solve for a Decentralized Equilibrium (DE) for any feasible monetary and fiscal policy. In this DE, (i) all households maximize utility (ii) a fraction \((1-\theta)\) of firms maximize profits by choosing the identical price \( P^g_t \), while the rest, \( \theta \), set their previous period prices (iii) all constraints are satisfied and (iv) all markets clear (details are available upon request).

The DE can be summarized by the following equilibrium conditions (all quantities are in per capita terms):

\[
c_t^{-\sigma} = \beta E_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau^g_t)} \left[ (1 + \frac{\tau^k_t}{\tau^g_t}) r^k_{t+1} + (1 - \delta) \right]
\]

(146)

\[
c_t^{-\sigma} = \beta E_t R_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau^g_t)} \frac{P_t}{P_{t+1}}
\]

(147)

\[
\chi_{nt}m^{-\mu}_t - \frac{c_t^{-\sigma}}{(1 + \tau^g_t)} + \beta E_t \frac{c^{-\sigma}_{t+1}}{(1 + \tau^g_{t+1})} \frac{P_t}{P_{t+1}} = 0
\]

(148)
\[ x_n - \frac{n^n}{c^n} = \frac{(1 - \tau^n)}{(1 + \tau^n)} w_t \]

\[ k_t = (1 - \delta) k_{t-1} + x_t \]

\[ \sum_{k=0}^{\infty} \theta^k E_t \left[ \frac{P_{t+k}^*}{P_t} \right]^{-\phi} y_{t+k} \left( \frac{P_{t+k}^*}{P_t} - \frac{\phi}{\phi-1} mc_{t+k} \frac{P_{t+k}}{P_t} \right) = 0 \]

\[ w_t = mc_t (1 - a) \frac{y_t}{n_t} \]

\[ r_t^k = mc_t \frac{y_t}{k_t} \]

\[ d_t = y_t - w_t n_t - r_t^k k_{t-1} \]

\[ y_t = \frac{1}{(\frac{\bar{P}}{P})^{-\phi}} A_t k_{t-1}^{a} n_t^{1-a} \]

\[ b_t + m_t = R_{t-1} b_{t-1} + \frac{P_{t-1}}{P_t} + m_{t-1} \frac{P_{t-1}}{P_t} g_t - \tau_t c_t - \tau^n t w_t n_t - \tau_t r_t^k k_{t-1} + d_t - \tau_t^l \]

\[ y_t = c_t + x_t + g_t \]

\[ (P_t)^{1-\phi} = \theta (P_{t-1})^{1-\phi} + (1 - \theta) \left( \frac{P^*}{P_t} \right)^{1-\phi} \]

\[ (\bar{P}_t)^{-\phi} = \theta (\bar{P}_{t-1})^{-\phi} + (1 - \theta) \left( \frac{\bar{P}^*}{\bar{P}_t} \right)^{-\phi} \]

where \( \bar{P}_t \equiv \left( \sum_{b=1}^{N} [P_t (h)]^{-\phi} \right)^{-\frac{1}{b}} \) and \( (\frac{\bar{P}}{P})^{-\phi} \) is a measure of price dispersion.

We thus have 14 equilibrium conditions for the DE. To solve the model, we need to specify the policy regime and thus classify policy instruments into endogenous and exogenous. Regarding the conduct of monetary policy, we assume that the nominal interest rate, \( R_t \), is used as a policy instrument, while, regarding fiscal policy, we assume that the residually determined public financing policy instrument is the end-of-period public debt, \( b_t \). Then, the 14 endogenous variables are \( \{ y_t, c_t, n_t, x_t, k_t, m_t, b_t, P_t, P^*_t, P_t, w_t, mc_t, dt, \} \). This is given the independently set policy instruments, \( \{ R_t, r_t^*, \tau_t^*, \tau_t^n, g_t, \} \), technology, \( \{ A_t \} \), and initial conditions for the state variables.

3.2.6 Decentralized equilibrium transformed (for any feasible policy)

Before we specify the motion of independently set policy instruments and exogenous stochastic variables, we rewrite the above equilibrium conditions, first, by using inflation rates rather than price levels, second, by writing the firm’s optimality condition (151) in recursive form and, third, by introducing a new equation that helps us to compute expected discounted lifetime utility. Details for each step are available upon request.
Variables expressed in ratios. We define three new endogenous variables, which are the gross inflation rate $\Pi_t \equiv \frac{P_t}{P_{t-1}}$, the auxiliary variable $\Theta_t \equiv \frac{P_{t}^{m}}{P_{t}}$, and the price dispersion index $\Delta_t \equiv \left[\frac{\Pi_t}{P_t}\right]^{-\phi}$. We also find it convenient to express the two exogenous fiscal spending policy instruments as ratios of GDP, $s_t^g \equiv \frac{s_t}{y_t}$ and $s_t^l \equiv \frac{s_t}{y_t}$.

Thus, from now on, we use $\Pi_t, \Theta_t, \Delta_t, s_t^g, s_t^l$ instead of $P_t, P_{t}^{m}, P_{t}^{l}, g_t, r_t$ respectively.

Equation (151) expressed in recursive form. Following Schmitt-Grohé and Uribe (2007), we look for a recursive representation of (151):

$$\sum_{k=0}^{\infty} (\theta)^k E_t \Xi_{t,t+k} \left[\frac{P_{t+1}^{m}}{P_{t+1}}\right]^{-\phi} y_{t+k} \left\{\frac{P_{t}^{m}}{P_t} - \frac{\phi}{(\phi - 1) m c_{t+k}} \frac{P_{t+k}}{P_t}\right\} = 0$$

(160)

We define two auxiliary endogenous variables:

$$z_1^1 \equiv \sum_{k=0}^{\infty} (\theta)^k E_t \Xi_{t,t+k} \left[\frac{P_{t+1}^{m}}{P_{t+1}}\right]^{-\phi} y_{t+k} \frac{P_{t+1}^{m}}{P_t}$$

(161)

$$z_1^2 \equiv \sum_{k=0}^{\infty} (\theta)^k E_t \Xi_{t,t+k} \left[\frac{P_{t+1}^{m}}{P_{t+1}}\right]^{-\phi} y_{t+k} m c_{t+k} \frac{P_{t+k}}{P_t}$$

(162)

Using these two auxiliary variables, $z_1^1$ and $z_1^2$, we come up with two new equations which enter the dynamic system and allow a recursive representation of (160). In particular, we can replace equilibrium equation (160) with:

$$z_1^1 = \frac{\phi}{(\phi - 1)} z_1^2$$

(163)

where:

$$z_1^1 = \Theta_t \frac{1}{1 - \phi} y_t + \beta \theta E_t \frac{\tau_{t+1}^{c}\left[\frac{\Theta_t}{\Theta_{t+1}}\right]^{-\phi - 1} \left(\frac{1}{\Pi_{t+1}}\right)^{-\phi}}{1 + \tau_{t+1}^{c}} z_1^1$$

(164)

$$z_1^2 = \Theta_t \frac{1}{1 - \phi} y_t m c_t + \beta \theta E_t \frac{\tau_{t+1}^{c}\left[\frac{\Theta_t}{\Theta_{t+1}}\right]^{-\phi - 1} \left(\frac{1}{\Pi_{t+1}}\right)^{-\phi}}{1 + \tau_{t+1}^{c}} z_1^2$$

(165)

Thus, from now on, we use (163), (164) and (165) instead of (160).

Lifetime utility written as a first-order dynamic equation. To compute social welfare, we follow Schmitt-Grohé and Uribe (2007) by defining a new endogenous variable, $V_t$, whose motion is:

$$V_t = \frac{c_t^{1-\sigma}}{1 - \sigma} - \frac{\beta E_t \xi_{t+1}^{1+\phi}}{1 + \phi} + \frac{\beta E_t \xi_{t+1}^{1-\mu}}{1 - \mu} + \frac{\beta E_t \xi_{t+1}^{1-\zeta}}{1 - \zeta}$$

(166)

where $V_t$ is the expected discounted lifetime utility of the household at any $t$.

Thus, from now on, we add equation (166) and the new variable $V_t$ to the equilibrium system.
3.2.7 Policy rules

Following the related New Keynesian literature, we focus on simple rules meaning that the monetary and fiscal authorities react to a small number of easily observable macroeconomic indicators. In particular, we allow the nominal interest rate, \( R_t \), to follow a rather standard Taylor rule meaning that it can react to inflation and the output gap, while we allow the non-lump sum spending-tax policy instruments, \( s_t^g, \tau_t^c, \tau_t^k, \tau_t^n \), to react to the public debt burden and the output gap. Finally, we allow all policy instruments to also have a stochastic part which captures unexpected discretionary changes in policy. In particular, following e.g. Schmitt-Grohé and Uribe (2007), we use policy rules of the form:

\[
\log \left( \frac{R_t}{\bar{R}} \right) = \phi_x \log \left( \frac{\Pi_t}{\Pi} \right) + \phi_y \log \left( \frac{y_t}{y} \right) + \nu_t^R \\
\]

(167)

\[
s_t^g - s^g = -\gamma_t^g (l_{t-1} - l) - \gamma_t^y (y_t - y) + \nu_t^g \\
\]

(168)

\[
\tau_t^c - \tau^c = \gamma_t^c (l_{t-1} - l) + \gamma_t^y (y_t - y) + \nu_t^c \\
\]

(169)

\[
\tau_t^k - \tau^k = \gamma_t^k (l_{t-1} - l) + \gamma_t^y (y_t - y) + \nu_t^k \\
\]

(170)

\[
\tau_t^n - \tau^n = \gamma_t^n (l_{t-1} - l) + \gamma_t^y (y_t - y) + \nu_t^n \\
\]

(171)

where variables without time subscripts denote long-run values, \( \phi_x, \phi_y, \gamma_t^g, \gamma_t^y \geq 0 \), where \( q_t \equiv (s_t^g, \tau_t^c, \tau_t^k, \tau_t^n) \), are feedback policy coefficients, \( l_{t-1} \equiv \frac{R_{t-1} \Pi_{t-1}}{y_{t-1}} \) denotes the inherited public debt burden as share of GDP and \( \nu_t^R, \nu_t^g, \nu_t^c, \nu_t^k, \nu_t^n \) are exogenous stochastic variables (defined below).

3.2.8 Exogenous stochastic variables

We assume that all the exogenous stochastic variables follow AR(1) processes:

\[
\log A_t = (1 - \rho^A) \log (A) + \rho^A \log A_{t-1} + \varepsilon_t^A \\
\]

(172)

\[
\log \nu_t^R = (1 - \rho^R) \log (\nu^R) + \rho^R \log \nu_{t-1}^R + \varepsilon_t^R \\
\]

(173)

\[
\log \nu_t^g = (1 - \rho^g) \log (\nu^g) + \rho^g \log \nu_{t-1}^g + \varepsilon_t^g \\
\]

(174)

\[
\log \nu_t^c = (1 - \rho^c) \log (\nu^c) + \rho^c \log \nu_{t-1}^c + \varepsilon_t^c \\
\]

(175)

\[
\log \nu_t^k = (1 - \rho^k) \log (\nu^k) + \rho^k \log \nu_{t-1}^k + \varepsilon_t^k \\
\]

(176)

\[
\log \nu_t^n = (1 - \rho^n) \log (\nu^n) + \rho^n \log \nu_{t-1}^n + \varepsilon_t^n \\
\]

(177)

where \( 0 \leq \rho^i \leq 1 \) are persistence parameters and \( \varepsilon_t^i \sim N \left( 0, \sigma_t^2 \right) \) where \( i = A, R, g, c, k, n \).
3.2.9 Summing up

Using all the above, the final non-linear stochastic equilibrium system is:

\[
\frac{c_t}{R_t} = \frac{c_{t+1}^{1-\sigma}}{(1 + \tau_{t+1}^{c})} \left[ (1 - \tau_{t+1}^{c}) r_{t+1}^{k} + (1 - \delta) \right] 
\]

(178)

\[
\frac{c_t^{-\sigma}}{R_t} \frac{1}{(1 + \tau_{t}^{c})} = \beta E_{t} \frac{c_{t+1}^{1-\sigma}}{(1 + \tau_{t+1}^{c})} \frac{1}{\Pi_{t+1}} 
\]

(179)

\[
\chi_{m} m_t^{-\mu} = \frac{c_{t}^{1-\mu}}{(1 + \tau_{t}^{c})} + \beta E_{t} c_{t+1}^{1-\mu} \frac{1}{\Pi_{t+1}} = 0 
\]

(180)

\[
\chi_{n} \frac{n_{t}}{c_{t}} = (1 - \tau_{t}^{n}) \frac{w_{t}}{(1 + \tau_{t}^{c})} 
\]

(181)

\[
k_{t} = (1 - \delta) k_{t-1} + x_{t} 
\]

(182)

\[
z_{t}^{1} = \frac{\phi - 1}{\phi} z_{t}^{2} 
\]

(183)

\[
w_{t} = mc_{t}(1 - \alpha) \frac{y_{t}}{n_{t}} 
\]

(184)

\[
r_{t}^{k} = mc_{t} \frac{y_{t}}{k_{t-1}} 
\]

(185)

\[
d_{t} = y_{t} - w_{t}n_{t} - r_{t}^{k}k_{t-1} 
\]

(186)

\[
y_{t} = \frac{1}{\Delta_{t}} A_{t} k_{t-1}^{0} n_{t-1}^{1-a} 
\]

(187)

\[
b_{t} + m_{t} = R_{t-1} b_{t-1} \frac{1}{\Pi_{t}} + m_{t-1} \frac{1}{\Pi_{t}} + s_{t}^{2} y_{t} - \tau_{t}^{c} c_{t} - \tau_{t}^{n} w_{t} n_{t} - \tau_{t}^{k} r_{t}^{k} k_{t-1} + d_{t} - \tau_{t}^{l} 
\]

(188)

\[
y_{t} = c_{t} + x_{t} + s_{t}^{2} y_{t} 
\]

(189)

\[
\Pi_{t}^{1-\phi} = \theta + (1 - \theta) [\Theta_{t} \Pi_{t}]^{1-\phi} 
\]

(190)

\[
\Delta_{t} = (1 - \theta) \Theta_{t}^{-\phi} + \theta \Pi_{t}^{\phi} \Delta_{t-1} 
\]

(191)

\[
z_{t}^{1} = y_{t} \Theta_{t}^{-\phi-1} + \beta \theta E_{t} c_{t+1}^{1-\sigma} \frac{1 + \tau_{t}^{c}}{c_{t}^{\sigma}} \left( \frac{\Theta_{t}}{\Theta_{t+1}} \right)^{-\phi-1} \Pi_{t+1}^{\phi} z_{t+1}^{1} 
\]

(192)

\[
z_{t}^{2} = \Theta_{t}^{-\phi} y_{t} mc_{t} + \beta \theta E_{t} c_{t+1}^{1-\sigma} \frac{1 + \tau_{t}^{c}}{c_{t}^{\sigma}} \left( \frac{\Theta_{t}}{\Theta_{t+1}} \right)^{-\phi} \Pi_{t+1}^{\phi-1} z_{t+1}^{2} 
\]

(193)

\[
V_{t} = \frac{c_{t}^{1-\sigma}}{1 - \sigma} + \chi_{m} \frac{m_{t}^{1-\mu}}{1 - \mu} - \chi_{n} \frac{n_{t}^{1+\phi}}{1 + \phi} + \chi_{y} \frac{(s_{t}^{2} y_{t})^{1-\zeta}}{1 - \zeta} + \beta E_{t} V_{t+1} + \phi_{z} \log \left( \frac{R_{t}}{R} \right) = \phi_{x} \log \left( \frac{\Pi_{t}}{\Pi} \right) + \phi_{y} \log \left( \frac{y_{t}}{y} \right) + \nu_{t}^{R} 
\]

(194)

\[
\log \left( \frac{R_{t}}{R} \right) = \phi_{x} \log \left( \frac{\Pi_{t}}{\Pi} \right) + \phi_{y} \log \left( \frac{y_{t}}{y} \right) + \nu_{t}^{R} 
\]

(195)
There are therefore 23 equations in 23 endogenous variables, \( \{y_t, c_t, k_t, m_t, b_t, \Pi_t, \Theta_t, \Delta_t, w_t, mc_t, d_t, r_t^k, z_t^1, z_t^2, V_t, R_t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n, l_t\} \). Among them, there are 17 non-predetermined or jump variables, \( \{y_t, c_t, n_t, x_t, \Pi_t, \Theta_t, \Delta_t, w_t, mc_t, d_t, r_t^k, z_t^1, z_t^2, V_t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n\} \), and 6 predetermined or state variables, \( \{R_t, b_t, m_t, \Delta_t, l_t\} \). This is given technology and policy shocks, \( \{A_t, \nu_t^y, \nu_t^c, \nu_t^k, \nu_t^n\} \), and initial conditions for the state variables. To solve this first-order non-linear difference equation system, we will take a second-order approximation around its long-run solution (details are in subsection 3.4.1 below). We first need to solve for the long run.

### 3.3 Data, calibration and long-run solution

This section solves numerically for the long run of the above model economy by using data from the euro zone. Since money is neutral in the long-run, interest rate policy does not matter to the real economy in the long run. Also, since fiscal policy instruments react to deviations of macroeconomic indicators from their long-run values, feedback fiscal policy coefficients do not play any role in the long run.

#### 3.3.1 Data and calibration

The data are from OECD Economic Outlook no. 89. The time unit is meant to be a quarter. Our parameter values are summarized in Table 1.
Table 1: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.33</td>
<td>share of capital</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9926</td>
<td>discount factor</td>
</tr>
<tr>
<td>$\mu$</td>
<td>3.42</td>
<td>real money balances elasticity</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.021</td>
<td>capital depreciation rate (quarterly)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>6</td>
<td>price elasticity of demand</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1</td>
<td>Frisch labour supply elasticity</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>elasticity of intertemporal substitution</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>1</td>
<td>elasticity of public consumption in utility</td>
</tr>
<tr>
<td>$\theta$</td>
<td>2/3</td>
<td>share of firms which cannot reset their prices</td>
</tr>
<tr>
<td>$\chi_m$</td>
<td>0.05</td>
<td>preference parameter for real money balances</td>
</tr>
<tr>
<td>$\chi_n$</td>
<td>6</td>
<td>preference parameter for hours worked</td>
</tr>
<tr>
<td>$\chi_g$</td>
<td>0.1</td>
<td>preference parameter for public good</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.8</td>
<td>serial correlation of TFP shock</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>0.85</td>
<td>serial correlation of monetary shock</td>
</tr>
<tr>
<td>$\rho^s$</td>
<td>0.87</td>
<td>serial correlation of spending shock</td>
</tr>
<tr>
<td>$\rho^c$</td>
<td>0.96</td>
<td>serial correlation of consumption tax shock</td>
</tr>
<tr>
<td>$\rho^K$</td>
<td>0.97</td>
<td>serial correlation of capital tax shock</td>
</tr>
<tr>
<td>$\rho^n$</td>
<td>0.94</td>
<td>serial correlation of labour tax shock</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.0062</td>
<td>standard deviation of innovation to TFP shock</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>0.005</td>
<td>standard deviation of innovation to monetary shock</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>0.016</td>
<td>standard deviation of innovation to spending shock</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>0.001</td>
<td>standard deviation of innovation to consumption tax shock</td>
</tr>
<tr>
<td>$\sigma_k$</td>
<td>0.003</td>
<td>standard deviation of innovation to capital tax shock</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>0.0005</td>
<td>standard deviation of innovation to labour tax shock</td>
</tr>
</tbody>
</table>

The value of the rate of time preference, $\beta$, follows from $R = 1.0075$, which is the average gross nominal interest rate in the data, and from setting $\Pi = 1$ for the long-run gross inflation rate. The real money balances elasticity, $\mu$, is taken from Pappa and Neiss (2005). The elasticity of intertemporal substitution, $\sigma$, the Frisch labour elasticity, $\eta$, and the price elasticity of demand, $\phi$, are as in Andrés and Doménech (2006) and Gali (2008). Regarding the preference parameters in the utility function, $\chi_m$ is chosen so as to obtain a yearly steady-state value of real money balances as ratio of output equal to 1.97 (0.5) quarterly (annually), $\chi_n$ is chosen so as to obtain steady-state labour hours equal to 0.28, while $\chi_g$ is set at 0.1 which is a common valuation of public goods in related utility functions.

Concerning the exogenous stochastic variables, we start by setting $\rho_A = 0.8$ and $\sigma_A = 0.0062$ for the persistence parameter and the standard deviation respectively of TFP in equation (172), which are as in Andrés and Doménech (2006). Regarding the public spending and interest rate policy shocks in equations (174) and (173), we follow Bi et al. (2010) by setting $\rho^s = 0.87$ and $\rho^R = 0.85$ for their persistence parameters, and $\sigma_A = 0.016$ and $\sigma_R = 0.005$ for their standard deviations, respectively. Finally, we run OLS regressions for consumption, capital and labor tax rates using Euro-zone data from 2001-2010, which imply $\rho^c = 0.96$, $\rho^K = 0.97$ and $\rho^n = 0.94$ for the persistence parameters, and $\sigma_c = 0.001$, $\sigma_k = 0.003$ and $\sigma_n = 0.0005$ for the standard deviations in (175), (176) and (177) respectively.

The long-run values of the exogenous policy instruments, $\tau^c_t$, $\tau^K_t$, $\tau^n_t$, $s^c_t$, $s^K_t$, $b_t$, are either set at their data averages, or are calibrated to deliver data-consistent steady-state values for the endogenous variables. In particular, $\tau^c$, $\tau^K$, $\tau^n$ are the averages of the effective tax rates in the
data. We set lump-sum taxes, $s^l$, so as to get a value of 0.43 for the sum $-s^l + s^g$, when the public debt-to-output ratio is 3.4 quarterly (or 0.85 annually) as in the average data over the recent period 2008-2011. The long-run values of policy instruments are summarized in Table 2.

<table>
<thead>
<tr>
<th>Table 2: Long-run values of policy instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
</tr>
<tr>
<td>1.0075</td>
</tr>
</tbody>
</table>

3.3.2 Status quo long-run solution

Table 3 reports the long-run solution of the model economy presented in subsection 3.3, when we use the parameter values in Table 1 and the policy instruments in Table 2. For comparison with the actual economy, Table 3 also presents some key ratios in the data whenever available. The solution makes sense and the resulting great ratios are close to their values in the data. In the next sections, we will depart from this status quo long-run solution to study various policy experiments.

<table>
<thead>
<tr>
<th>Table 3: Long-run solution and some data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables</td>
</tr>
<tr>
<td>$y$</td>
</tr>
<tr>
<td>$c$</td>
</tr>
<tr>
<td>$n$</td>
</tr>
<tr>
<td>$x$</td>
</tr>
<tr>
<td>$k$</td>
</tr>
<tr>
<td>$m$</td>
</tr>
<tr>
<td>$b$</td>
</tr>
<tr>
<td>$\Pi$</td>
</tr>
<tr>
<td>$\Theta$</td>
</tr>
<tr>
<td>$\Delta$</td>
</tr>
<tr>
<td>$w$</td>
</tr>
<tr>
<td>$mc$</td>
</tr>
</tbody>
</table>

3.4 How we work

The aim of the paper is to study the implications of alternative policy rules for macroeconomic outcomes and lifetime utility. To make the comparison of alternative policies meaningful, we study optimal policy so that outcomes do not depend on ad hoc differences in the policy rules compared. As said, the welfare objective is household’s expected lifetime utility.

We will study two economic environments. In the first (see subsection 3.5), the role of economic policy is to stabilize the economy against temporary shocks as defined in subsection 3.2.8. This means that we depart from, and end up, at the same steady state as solved above. In the second environment (see subsection 3.6), the role of policy is twofold: to stabilize the economy against the same shocks as before and to improve resource allocation by gradually reducing the public debt ratio over time. Thus, in subsection 6, the transition dynamics will be driven, not only by shocks, but also by the difference between the initial pre-reformed steady state and the new reformed steady state (see also Cantore et al., 2012).
The reason we study two different economic environments (subsections 3.5 and 3.6) is that we want to see whether the welfare ranking of alternative policy rules changes when we move to a richer setup with a more ambitious policy as that in subsection 3.6. Also, within each economic environment, we will study optimal policy under relatively low and relatively high extrinsic uncertainty as measured by the standard deviation of the exogenous stochastic variables.

Irrespectively of the policy experiments chosen, we need to compute optimized policy rules and choose a criterion to welfare rank alternative policies. These are explained in what follows in the rest of this section.

3.4.1 How we compute optimized policy rules and the equilibrium

We work in two steps. In the first step, we search for the ranges of feedback policy coefficients as defined in (167-171) which allow us to get a locally determinate decentralized equilibrium (this is what Schmitt-Grohé and Uribe, 2007, call implementable rules). If necessary, these ranges will be further restricted so as to give meaningful solutions for the policy instruments (e.g. capital tax rates less than one). In this search for local determinacy, we experiment with one, or more, policy instruments and one, or more, operating targets at a time.

In the second step, within the determinacy ranges found above, we compute the welfare-maximizing values of feedback policy coefficients (this is what Schmitt-Grohé and Uribe, 2005 and 2007, call optimized policy rules). The welfare criterion is to maximize the conditional welfare, $V_0$, as defined in (166) above, where conditionality refers to the initial conditions chosen; the latter are given by the status quo long-run solution. To this end, following e.g. Schmitt-Grohé and Uribe (2004), we take a second-order approximation to both the equilibrium conditions and the welfare criterion. As is well known, this is consistent with risk-averse behavior on the part of economic agents and can also help to avoid possible spurious welfare results that may arise when one takes a second-order approximation to the welfare criterion combined with a first-order approximation to the equilibrium conditions (see e.g. Gali, 2008, pp. 110-111, Malley et al., 2009, and, for a recent review, Benigno and Woodford, 2012).

In other words, we compute the feedback policy coefficients so as to maximize the second-order approximation of conditional welfare subject to the second-order approximation of the decentralized equilibrium when the feedback policy coefficients are restricted to be within prespecified ranges delivering determinacy (see details on the computational algorithm used in Appendix C).

3.4.2 How we welfare rank alternative regimes

To welfare rank alternative policy regimes, we need to evaluate their welfare gains, or losses, relative to a reference regime. We find it natural to define the latter as the case in which policy is passive, in the sense that all policy instruments are held constant and equal to their steady-state values which are as in the data averages.

Let $V_s^*$ denote the value function under a stabilization regime $s$. Thus,

$$V_0^* = E_0 \sum_{t=0}^{\infty} \beta^t U (c_s^t, n_s^t, m_s^t, g_s^t)$$

where $c_s^t, n_s^t, m_s^t$ and $g_s^t$ are the equilibrium values of consumption, hours worked, real money balances and public spending under regime $s$. 

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Let also $V^r_t$ denote the value function under the reference or passive regime, $r$. Thus,

$$V^r_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t U(c^r_t, n^r_t, m^r_t, g^r_t)$$  \hspace{1cm} (202)$$

where $c^r_t, n^r_t, m^r_t$ and $g^r_t$ are the equilibrium values of consumption, hours worked, real money balances and public spending under passive policy.

Then, following most of the literature on policy reforms, we denote by $\xi$ the permanent consumption subsidy that the household would need under the reference regime $r$ so as to be as well off as under regime $s$. Using the model parameterization in Table 1 above, $\xi$ is approximately given by:

$$\xi \simeq (V^s_0 - V^r_0) (1 - \beta)$$  \hspace{1cm} (203)$$

so that if $\xi > 0$ (resp. $\xi < 0$), the agent is better off under $s$ (resp. under $r$). The stabilization regime with the highest value of $\xi$ will be the most preferred one.

### 3.5 Stabilization policy

We now study the implications of different policy rules when the economy is hit by the temporary shocks as defined in subsection 3.2.8 above. Thus, the government solves a stabilization problem only. Technically, this means that we depart from, and end up, at the same steady state as that found in section 3.3 above, so that transition dynamics are driven by shocks only. Recall that, along the transition path, nominal rigidities imply that money is not neutral, so that feedback interest-rate policy, as defined in equation (167), matters to the real economy. Also recall that, along the transition path, different feedback fiscal rules, as defined in equations (168)-(171), have different implications.

#### 3.5.1 Results under relatively low uncertainty

We start with conditions for local determinacy. We report that economic policy guarantees determinacy when the nominal interest rate reacts to inflation aggressively with $\phi_y > 1$, that is, when the Taylor principle is satisfied, and, at the same time, the fiscal policy instruments, $q_t \equiv (s^g_t, r^k_t, r^\pi_t, r^\gamma_t)$, react in general to public liabilities above a critical minimum value, $\gamma^g_t > \gamma^\pi_t > 0$. By contrast, the values of $\phi_y$ and $\gamma^g_t$, measuring respectively the reaction of interest rate policy and fiscal policy to the output gap, are not found to be critical to determinacy. The regions of feedback policy coefficients that guarantee determinacy are reported in detail in the notes of Table 4.\(^{12}\) Thus, the general message is that monetary and fiscal policy need to interact with each other in a specific way for policy to guarantee determinacy. Or, as Leeper (2010) puts it, there is a "dirty little secret": for monetary policy to control inflation, fiscal policy must behave in a particular manner.

**Optimized policy rules** Within the determinacy areas, we now turn to optimized policy rules. Thus, as explained above, we search for those values of the feedback policy coefficients, and the indicators that the policy instruments respond to, that maximize household’s conditional welfare

\(^{12}\)Actually, we can distinguish two regions of determinacy. In addition to the one discussed above, there is another region in which fiscal policy does not react to public liabilities, i.e. $\gamma^g_t = 0$ for all fiscal instruments, while monetary policy reacts to inflation mildly with $\phi_y < 1$. This region is welfare inferior to the region discussed above. It also contains some sub-areas where determinacy breaks down. Several other papers have distinguished between the same two areas of determinacy (e.g. Leeper, 1991, and Schmitt-Grohé and Uribe, 2007).
in (166), when we allow monetary policy to react to inflation and output, and fiscal policy to react to public liabilities and output. To understand the logic of our results, and following usual practice, we start by examining one fiscal instrument at a time. Results are reported in Table 4. The values of \( \xi \) give the welfare gains vis-a-vis the benchmark case without stabilization policy, namely, when all feedback policy coefficients are exogenously set to zero.

The main results are as follows. First, in terms of fiscal policy, the best instrument to use is government spending. The next best choice is to use the consumption tax rate, in turn, capital and, lastly, the labour tax rate. Notice that use of the labor tax rate is distinctly the worst choice to make.

Second, the best policy mix is to use government spending to react to debt only and the nominal interest rate to react to inflation only. In this case, with our baseline parameterization, a welfare gain of 2.2% is achieved when the monetary and fiscal authorities jointly intervene to stabilize the economy against shocks.

Third, in all cases, the monetary authority should react aggressively to inflation, \( \phi_x = 3 \), and the fiscal authorities should react to debt, \( \gamma^d_y > 0 \). This is consistent with the consensus assignment (see e.g. Kirsanova et al., 2009).

Fourth, policy reaction to the output gap is desirable, \( \phi_y, \gamma^o_y > 0 \), only when we use taxes for debt stabilization. This applies to both monetary and fiscal policy. The idea is that changes in taxes hurt the real economy, so, at the same time, monetary and fiscal policy need to be concerned about the output gap; by contrast, this is not necessary when we use public spending for debt stabilization, hence \( \phi_y = \gamma^o_y = 0 \) in this case. Therefore, the desirability of output stabilization, or counter-cyclical policy, depends on which fiscal policy instrument is used for debt stabilization. Notice the strong reaction of the labor tax rate to the output gap, \( \gamma^o_y \) (see below for details).

Fifth, fiscal reaction to public debt should be mild, except from the case in which we use the capital tax rate as a debt stabilization device (see the high value of \( \gamma^k_l \) in Table 4). The intuition behind this relatively strong reaction is as follows. When we use the capital tax rate to react to debt imbalances in the short run, this works like a capital levy on existing wealth which is not so distorting. At the same time, debt stabilization in the short run implies a reduced fiscal burden and expectations of lower capital taxes in the future, which can in turn stimulate investment. This dynamic effect is consistent with the theoretical results of Chamley (1986) and Judd (1985) and the simulations of Altig et al. (2001) who study tax reforms in the US. This is why \( \gamma^k_l \) is high (at least, when extrinsic macroeconomic volatility is relatively low - see also below).
### Table 4: Optimal monetary reaction to inflation and output and optimal fiscal reaction to debt and output\(^\text{13}\)

<table>
<thead>
<tr>
<th>Policy instruments</th>
<th>Optimal interest-rate reaction to inflation and output</th>
<th>Optimal fiscal reaction to debt and output</th>
<th>(\xi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_t) (s) (t)</td>
<td>(\phi_\pi = 3) (\phi_y = 0) (\gamma_t^g = 0.1) (\gamma_y^g = 0)</td>
<td>(\gamma_t^g = 0)</td>
<td>0.022</td>
</tr>
<tr>
<td>(R_t) (\tau) (t)</td>
<td>(\phi_\pi = 3) (\phi_y = 0.096) (\gamma_t^k = 0.1) (\gamma_y^k = 0.3428)</td>
<td>(\gamma_t^k = 0.3428)</td>
<td>0.0206</td>
</tr>
<tr>
<td>(R_t) (\tau) (t) (k)</td>
<td>(\phi_\pi = 3) (\phi_y = 0.39) (\gamma_t^n = 0.1) (\gamma_y^n = 0.051)</td>
<td>(\gamma_t^n = 0.051)</td>
<td>0.02</td>
</tr>
<tr>
<td>(R_t) (\tau) (t) (n)</td>
<td>(\phi_\pi = 3) (\phi_y = 0.044) (\gamma_t^n = 0.1) (\gamma_y^n = 3)</td>
<td>(\gamma_t^n = 3)</td>
<td>0.0195</td>
</tr>
</tbody>
</table>

Notes: When we use government spending, the ranges are \(\phi_\pi \in [1.1, 3]\), \(\phi_y \in [0, 3]\), \(\gamma_t^g \in [0.1, 3]\) and \(\gamma_y^g \in [0, 3]\). When we use consumption taxes, the ranges are \(\phi_\pi \in [1.1, 3]\), \(\phi_y \in [0, 3]\), \(\gamma_t^k \in [0.1, 3]\) and \(\gamma_y^k \in [0, 3]\). When we use capital taxes, the ranges are \(\phi_\pi \in [1.25, 3]\), \(\phi_y \in [0, 3]\), \(\gamma_t^n \in [0.1, 3]\) and \(\gamma_y^n \in [0, 3]\). When we use labor taxes, the ranges are \(\phi_\pi \in [1.25, 3]\), \(\phi_y \in [0, 3]\), \(\gamma_t^n \in [0, 3]\) and \(\gamma_y^n \in [0, 3]\).

In the above experiments, we switched on one fiscal instrument at a time. To further test our results, we now switch on all fiscal policy instruments at the same time. That is, the monetary authorities are free to use the nominal interest rate to react to both inflation and the output gap, while the fiscal authorities are allowed to use all spending-tax instruments simultaneously to react to both public liabilities and the output gap. We report that, in this case, the ranges of determinacy are much smaller than before when we used one fiscal instrument at a time. Nevertheless, despite this restriction, we report that our main results in Table 4 are not affected. Namely, the best mix is to use government spending to react to debt only and the nominal interest rate to react to inflation only.

**Impulse response functions** We now present the impulse response functions (IRFs) of some key endogenous variables when there is a negative TFP shock (with a relatively low standard deviation, 0.0062). As pointed out in the Introduction, since, in most cases, it is optimal for policy instruments to react to more than one indicator at the same time, the IRFs can show which reaction dominates.

We start with the case in which we use the best possible policy mix (see Table 4, row 1). That is, we use the nominal interest rate to react to price inflation only and the public spending share to react to public debt only, with feedback coefficients \(\phi_\pi = 3\) and \(\gamma_y^g = 0.1\) respectively. All other policy feedback coefficients are set at zero, which means that \(\tau_t^c\), \(\tau_t^k\) and \(\tau_t^n\) remain constant at their steady-state values (data averages). Thus, we use the optimized policy rules:

\[
\log \left( \frac{R_t}{R} \right) = 3 \log \left( \frac{\Pi_t}{\Pi} \right) \quad (204)
\]

\[
s^t_t = s^g = -0.1 \ast (t_{t-1} - t) \quad (205)
\]

which imply the IRFs shown in Figure 1.

\(^{13}\)In Appendix B, Table 7 is as Table 4, but firms face a Rotemberg-type nominal fixity.
As shown in Figure 1, an adverse TFP shock leads to a contraction in output, $y$, as expected. As a result, government liabilities as a ratio of output, $l$, rise. Under optimized rules, the fiscal authorities find it optimal to react to this rise in public liabilities by decreasing government spending, $s^g$. Notice that $s^g$ decreases for several periods and returns back to its steady-state value only when $l$ manages to fall. In the very short term, an adverse supply shock and the sharp fall in output lead to a fall in inflation, which is accompanied by a fall in the nominal interest rate via the optimized Taylor rule. But, very soon, the adverse supply shock leads to higher marginal costs and hence higher price inflation (this is a standard effect in the New Keynesian literature) which is now accompanied by a rise in the nominal interest rate again via the optimized Taylor rule. Higher inflation erodes the real value of government liabilities, $l$, which, jointly with the recovery of output, help the recovery in public spending in the medium run.

It is also interesting to present IRFs when, for some political-economy reason, policy is restricted to follow the sub-optimal policy mixes reported in Table 4, rows 2-4. The economy is hit by the same adverse TFP shock as above.

We start with the case in which the fiscal instrument is the consumption tax rate. This is the second row in Table 4. Thus, the optimized policy rules are:

\[
\log \left( \frac{R_t}{R} \right) = 3 \log \left( \frac{\Pi_t}{\Pi} \right) + 0.096 \log \left( \frac{yt}{y} \right) \tag{206}
\]

\[
\tau^c_t - \tau^c = 0.1 (l_{t-1} - l) + 0.34 (yt - y) \tag{207}
\]
which imply the IRFs shown in Figure 2. Before we discuss results, we present the IRFs for the other taxes too.

Figure 2: Impulse response functions to a negative TFP shock when the fiscal instrument is the consumption tax rate

![Figure 2: Impulse response functions to a negative TFP shock when the fiscal instrument is the consumption tax rate](image)

We continue with the case in which the fiscal instrument is the capital tax rate. This is the third row in Table 4. Thus, the optimized policy rules are:

\[
\log \left( \frac{R_t}{R} \right) = 3 \log \left( \frac{\Pi_t}{\Pi} \right) + 0.39 \log \left( \frac{y_t}{y} \right) 
\]  

\[
\tau^k_t - \tau^k = 3 \times (l_{t-1} - l) + 0.051 \times (y_t - y) 
\]

which imply the IRFs shown in Figure 3.
We finally study the case in which the fiscal instrument is the labor tax rate. This is the last row in Table 4. Thus, the optimized policy rules are:

\[
\log \left( \frac{R_t}{R} \right) = 3 \log \left( \frac{\Pi_t}{\Pi} \right) + 0.044 \log \left( \frac{y_t}{y} \right) 
\]  
(210)

\[
\tau^n_t - \tau^n = 0.1 \ast (l_{t-1} - l) + 3 \ast (y_t - y) 
\]  
(211)

which imply the IRFs shown in Figure 4.
Figure 4: Impulse response functions to a negative TFP shock when the fiscal instrument is the labour tax rate.

Inspection of Figure 3 reveals that when the economy is hit by an adverse TFP shock, which results in an increase in the public debt ratio, the fiscal authorities find it optimal to increase capital tax rates at impact so as to stabilize the public debt ratio. By contrast, inspection of Figures 2 and 4 reveals that, in case they use consumption or labor tax rates, they find it optimal to reduce these tax rates, rather than increase them, at impact. This is at the expense of a long lasting rise in public debt. In particular, in the case of labor taxes, only when the effects of the adverse TFP shock fade away, the government starts increasing the labor tax rate to stabilize the debt dynamics.

Thus, in the case of consumption, and especially labor, taxes, the immediate priority should be given to output, rather than to debt. This is because a rise in consumption, and especially labor, taxes is particularly damaging to an economy hit by an adverse supply shock. By contrast, this is not the case with capital taxes. Capital taxes, if they are imposed in the very short run, they work as a tax on wealth, or a capital levy, so that they are less distorting than other taxes, especially since high capital taxes in the very short allow low capital taxes in the near future, the expectation of which can stimulate investment.

Summing up the evidence from IRFs in this subsection, when extrinsic uncertainty is relatively low and the only role of policy is to stabilize the economy against shocks, the policy priority should be given to the stabilization of debt cycles in case we use government spending and capital taxes. By contrast, when we use consumption and labor taxes, the policy priority should be given to the stabilization of output cycles. This is because consumption and labor taxes are more distorting than government spending and capital levies, so we cannot afford to use them for debt imbalances when the economy is in a recession; instead, they should be spared to address the output cycle first and only, in turn, the debt cycle.
3.5.2 Results under relatively high uncertainty

So far, the best policy mix is to use public spending to react to public debt imbalances only, and the nominal interest rate to react to inflation only. We now check the robustness of this result, when we face a more volatile macroeconomic environment. In particular, other things equal, we arbitrarily increase the standard deviation of the TFP shock to 0.01 instead of 0.0062 that was used so far.

To save on space, we present the main results only and discuss what differs from the previous subsection 3.5.1 which assumed relatively low macro volatility. We report that the policy coefficient regions required for determinacy remain the same as in Table 4 above.

Optimized policy rules  The new results are reported in Table 5. Comparison of Tables 5 and 4 implies that the main results remain unchanged. For instance, the welfare ranking of fiscal policy instruments does not change. Also, in all cases, interest rate policy should react aggressively to inflation and fiscal policy should react to debt imbalances. But there are also some new results. In Table 5, all values of $\gamma^q_y$, where $q_t \equiv (s^q_t, \tau^k_t, \tau^n_t)$, are positive. Thus, in a more volatile environment, fiscal reaction to the output gap is productive whatever fiscal policy instrument we choose to use. By contrast, recall that $\gamma^q_y$ was zero in Table 4. Actually, in Table 5, fiscal reaction to the output gap should be stronger than fiscal reaction to public debt, i.e. $\gamma^q_l < \gamma^q_y$ in all cases. That is, in a more volatile environment, the fiscal priority should be given to the business cycle (this is confirmed below when we present impulse response functions).

Table 5: Optimal monetary reaction to inflation and output and optimal fiscal reaction to debt and output

<table>
<thead>
<tr>
<th>Policy instruments</th>
<th>Optimal interest-rate reaction to inflation and output</th>
<th>Optimal fiscal reaction to debt and output</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_t$ $s^q_t$</td>
<td>$\phi^\pi = 3$</td>
<td>$\gamma^q_l = 0.1$</td>
<td>$\gamma^q_y = 0.11$</td>
</tr>
<tr>
<td>$R_t$ $\tau^c_t$</td>
<td>$\phi^\pi = 3$</td>
<td>$\gamma^c_l = 0.1$</td>
<td>$\gamma^c_y = 0.28$</td>
</tr>
<tr>
<td>$R_t$ $\tau^k_t$</td>
<td>$\phi^\pi = 3$</td>
<td>$\gamma^k_l = 1.34$</td>
<td>$\gamma^k_y = 3$</td>
</tr>
<tr>
<td>$R_t$ $\tau^n_t$</td>
<td>$\phi^\pi = 3$</td>
<td>$\gamma^n_l = 0.1$</td>
<td>$\gamma^n_y = 3$</td>
</tr>
</tbody>
</table>

Notes: See the notes in Table 4 above.

Impulse response functions  As above, we present IRFs when the economy is hit by an adverse TFP shock. We start with the best possible policy mix in Table 5, row 1. Thus, the optimized rules for the nominal interest rate and the public spending share are:

$$\log \left( \frac{R_t}{R} \right) = 3 \log \left( \frac{\Pi_t}{\Pi} \right)$$  \hspace{1cm} (212)

$$s^q_t - s^q = -0.1 \times (l_{t-1} - l) - 0.11 \times (y_t - y)$$  \hspace{1cm} (213)

which imply the IRFs shown in Figure 5.
Before we discuss results, we also present IRFs under the suboptimal policy mixes in Table 5, rows 2-4. When the fiscal policy instrument is the consumption tax rate, we have the optimized rules:

\[
\log \left( \frac{R_t}{R_t} \right) = 3 \cdot \log \left( \frac{\Pi_t}{\Pi_t} \right) + 0.055 \cdot \log \left( \frac{y_t}{y} \right) + 0.1 \cdot (l_{t-1} - l) + 0.28 \cdot (y_t - y) \tag{214}
\]

\[
\tau_t^f - \tau_t^c = \log \left( \frac{\Pi_t}{\Pi_t} \right) + 0.1 \cdot (l_{t-1} - l) + 0.28 \cdot (y_t - y) \tag{215}
\]

which imply the IRFs shown in Figure 6.
When the fiscal policy instrument is the capital tax rate, we have the optimized rules:

$$\log \left( \frac{R_t}{R} \right) = 3 \times \log \left( \frac{H_t}{H} \right) + 0.22 \times \log \left( \frac{y_t}{y} \right)$$  \hspace{1cm} (216)

$$\tau_c^t - \tau_c^k = 1.34 \times (l_t - l) + 3 \times (y_t - y)$$ \hspace{1cm} (217)

which imply the IRFs shown in Figure 7.
When the fiscal policy instrument is the labor tax rate, we have the optimized rules:

\[
\log\left(\frac{R_t}{R}\right) = 3 \log\left(\frac{H_t}{H}\right) + 0.05 \log\left(\frac{y_t}{y}\right)
\]  \hspace{1cm} (218)

\[
\tau^n_t - \tau^n = 0.1 \times (l_{t-1} - l) + 3 \times (y_t - y)
\]  \hspace{1cm} (219)

which imply the IRFs shown in Figure 8.
Figures 5-8 imply that public spending should rise, and tax rates should fall, at impact. In other words, when the economy is hit by a relatively strong adverse shock, the immediate reaction of fiscal authorities should be to counter the recession by following an expansionary fiscal policy and only in turn address debt imbalances. Therefore, the difference between Figures 1-4 and Figures 5-8 is that in the latter, which describe the case of relatively high uncertainty, all fiscal instruments should give priority to the business cycle in the short run, while, in the former, which described the case of relatively low uncertainty, this applied only to the more distorting policy instruments (consumption and labor taxes).

3.6 Stabilization and resource allocation policy together

We now study the implications of different policy rules when the economy is not only hit by temporary shocks as in the previous section but the government also wants to reduce the GDP share of public debt over time. In particular, we assume that the government reduces this share from 85% (which is its average value in the data over the last years - see section 3.3) to 60% (we choose the value of 60% simply because it is the reference rate of the Maastricht Treaty). Debt consolidation allows, other things equal, a cut in the tax rates, and a rise in public spending, in the long run, although this comes at the cost of higher taxes and lower public spending during the early phase of the transition path. Public financing issues, and how we model debt consolidation, are explained in the following subsection.
3.6.1 How we model debt consolidation

It is well recognized that the implications of fiscal reforms, like debt consolidation, depend heavily on the public financing policy instrument used, namely, which policy instrument adjusts endogenously to accommodate the exogenous changes in fiscal policy (see e.g. Leeper et al., 2010, and Leeper, 2010). Here, we will assume that, along the transition path, fiscal reforms are accommodated by adjustments in fiscal policy instruments, namely, the share of public spending, and the tax rates on capital income, labour income and consumption. To understand the logic of our results, and following usual practice in related studies, we will experiment with one fiscal instrument at a time. This means that, along the transition path, we allow one of the fiscal policy instruments to react to public debt imbalances, so as to stabilize debt around its new target value of 0.60 and, at the same time, the same fiscal policy instrument adjusts residually in the long run to close the government budget. Thus, the policy rules for these instruments are as in subsection 3.2.7 above except that now the targetted, or long-run, values are those of the reformed long-run equilibrium. All other fiscal policy instruments, except the one used for stabilization, remain unchanged and equal to the pre-reform steady-state values. The feedback policy coefficients of the fiscal policy instrument used for stabilization along the transition path, as well as the feedback policy coefficients of the nominal interest rate, are chosen optimally as explained in subsection 3.4.1 and as we did in the previous section.

In particular, we work as follows. We first solve and compare the long-run equilibria with and without debt consolidation. In turn, setting, as initial conditions for the state variables, their steady state solution of the economy without debt consolidation (see the status quo long-run solution in subsection 3.3), we compute the equilibrium transition path of each reformed economy under optimized policy rules and thus calculate the associated discounted lifetime utility of the household. This is for each method of public financing. This utility is finally compared to its associated value if there was no stabilization at all. Thus, the reference regime is the same as that used in the previous section so that welfare comparisons are easy to make. Recall that the model is stochastic so that now there are two sources of transitional dynamics: temporary shocks and the difference between the initial and the new reformed steady state.

3.6.2 Results under relatively low uncertainty

We report that the determinacy areas remain as above. Nevertheless, in the policy rule for the capital tax rate, we need to restrict the feedback coefficient on public debt in order to get a meaningful capital tax rate less than one, \( \gamma^k_t < 1 \). In particular, the range is now narrower, \( \gamma^k_t \in [0.1, 0.2] \), than it was before, \( \gamma^k_t \in [0, 3] \). This can be explained by the Chamley-Judd result: when debt consolidation is among the policy aims, the fiscal authorities have an incentive to impose a high capital levy in the beginning of the time horizon to minimize the distortions during the rest of the transition period. The ranges of all feedback policy coefficients are summarized in the notes of Table 6.

Optimized policy rules The new results are reported in Table 6. Comparison of Table 6 to Tables 5 and 4 above implies that again the main results remain unchanged. For instance, the welfare ranking of fiscal policy instruments does not change. Also, in all cases, interest rate policy should react aggressively to inflation and fiscal policy should react to debt imbalances. But there are also some new results. The welfare gains from policy intervention are higher than in Tables 4 and 5 above except in the case of labor taxes. That is, other things equal, reducing the public debt, in addition to stabilizing the economy against shocks, is welfare improving except when we have to use a particularly distorting policy instrument like labor taxes. Also notice, in Table 6, that reaction to the output gap is recommended to both the fiscal and monetary authorities...
when they use the best possible mix, $R_t$ and $s^q_t$. The idea is that the effort to reduce public debt over time hurts the real economy, so, at the same time, monetary and fiscal policy need to be concerned about the output gap; by contrast, this was not necessary when the concern of policy was stabilization of shocks only (see Tables 4 and 5 above).

Table 6: Optimal monetary reaction to inflation and output and optimal fiscal reaction to debt and output

<table>
<thead>
<tr>
<th>Policy instruments</th>
<th>Optimal interest-rate reaction to inflation</th>
<th>Optimal fiscal reaction to debt and output</th>
<th>$\xi^{14}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_t$, $s^q_t$</td>
<td>$\phi_\pi = 3$</td>
<td>$\gamma^q_1 = 0.1$</td>
<td>0.0245</td>
</tr>
<tr>
<td></td>
<td>$\phi_\eta = 0.15$</td>
<td>$\gamma^q_2 = 0.17$</td>
<td></td>
</tr>
<tr>
<td>$R_t$, $\tau^c_t$</td>
<td>$\phi_\pi = 3$</td>
<td>$\gamma^c_1 = 0.16$</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>$\phi_\eta = 0.14$</td>
<td>$\gamma^c_2 = 0.34$</td>
<td></td>
</tr>
<tr>
<td>$R_t$, $\tau^p_t$</td>
<td>$\phi_\pi = 3$</td>
<td>$\gamma^p_1 = 0.215$</td>
<td>0.0229</td>
</tr>
<tr>
<td></td>
<td>$\phi_\eta = 0.49$</td>
<td>$\gamma^p_2 = 0$</td>
<td></td>
</tr>
<tr>
<td>$R_t$, $\tau^n_t$</td>
<td>$\phi_\pi = 3$</td>
<td>$\gamma^n_1 = 0.1$</td>
<td>0.0168</td>
</tr>
<tr>
<td></td>
<td>$\phi_\eta = 0.14$</td>
<td>$\gamma^n_2 = 3$</td>
<td></td>
</tr>
</tbody>
</table>

Notes: As in Table 4 except that now $\gamma^p_t \in [0.1, 0.2]$ in case we use capital taxes.

Impulse response functions  When we use the best policy mix in Table 6, row 1, the optimized policy rules are:

$$
\log \left( \frac{R_t}{R} \right) = 3 \log \left( \frac{\Pi_t}{\Pi} \right) + 0.15 \log \left( \frac{y_t}{y} \right) 
$$  
(220)

$$
s^q_t - s^q = 0.1 \left( l_t - l \right) + 0.17 \left( y_t - y \right) 
$$  
(221)

which imply the IRFs shown in Figure 9.

\[14\] In this case $\xi \approx \left[ \tilde{V}_0^* + V^* - \left( \tilde{V}_0^* + V^* \right) \right] \left( 1 - \beta \right)$. Where the reference regime is the passive regime under low uncertainty.

\[15\] Due to feasibility reasons we restrict the value of the feedback policy coefficient associated with public debt to be $0.1 < \gamma^p_t < 0.2$.  

55
Before we discuss results, we also present impulse response functions under the suboptimal policy mixes in Table 6, rows 2-4. When the fiscal policy instrument is the consumption tax rate, we have the optimized policy rules:

\[
\log \left( \frac{R_t}{R} \right) = 3 \log \left( \frac{\Pi_t}{\Pi} \right) + 0.14 \log \left( \frac{y_t}{y} \right)
\]  

(222)

\[
\tau_t^c - \tau_c^e = 0.14 \left( l_{t-1} - l \right) + 0.34 \left( y_t - y \right)
\]  

(223)

which imply the IRFs shown in Figure 10.
When the fiscal policy instrument is the capital tax rate, we have the optimized rules:

\[
\log \left( \frac{R_t}{R} \right) = 3 \log \left( \frac{\Pi_t}{\Pi} \right) + 0.49 \log \left( \frac{y_t}{y} \right)
\]

(224)

\[
\tau_t^k - \tau^k = 0.2 \ast (l_{t-1} - l)
\]

(225)

which imply the IRFs shown in Figure 11.
Finally, when the fiscal policy instrument is the labor tax rate, we have the optimized rules:

\[
\log \left( \frac{R_t}{R} \right) = 3 \log \left( \frac{\Pi_t}{\Pi} \right) + 0.14 \log \left( \frac{y_t}{\bar{y}} \right) \quad (226)
\]

\[
\tau_t^n - \tau^n = 0.1 (l_{t-1} - l) + 3 (y_t - y) \quad (227)
\]

which imply the IRFs shown in Figure 12.
Figure 12: Impulse responce functions to a negative TFP shock when the fiscal instrument is the labour tax rate

Figures 9-12 imply that public spending should fall, and tax rates should rise. In other words, the concern for debt consolidation more than offsets the concern for shock stabilization even when the economy is hit an adverse shock. This is the opposite from Figures 5-8 above. In Figures 5-8, all fiscal instruments should give priority to the business cycle at impact, and only then should be used to address debt imbalances. By constrast, in Figures 9-12, all fiscal instruments should be earmarked for the reduction in public debt all the time.

Finally, inspection of the above IRFs implies that the duration of the debt consolidation phase, and so the speed of debt reduction, depend on which fiscal instrument we use. If we use the public spending ratio, $s^g_t$, it is optimal to reduce the debt ratio from 85% to 60% within 40 quarters, if we use consumption taxes, $\tau^c_t$, within 50 quarters, if we use capital taxes, $\tau^k_t$, within 40 quarters and, finally, if we use labor taxes, $\tau^n_t$, in more than 100 quarters. Thus, the more distorting is the instrument used, the longer the period of adjustment should be.

3.6.3 Results under relatively high uncertainty

We report that, when we move to a more volatile economy as that defined in subsection 3.5.2, all qualitative results remain as in the previous subsection, 3.6.2. Thus, the results are driven by debt consolidation rather than by shock stabilization even in a more volatile environment.

3.7 Concluding remarks

This paper studied the optimal mix of monetary and fiscal policy actions in a New Keynesian model of a closed economy. The aim has been to welfare rank different fiscal (tax and spending)
policy instruments when the central bank can follow a Taylor rule for the interest rate. We did so in two policy environments: first, when the policy task was to stabilize the economy against shocks and, second, when the government faced two tasks, shock stabilization and debt consolidation.

Since the results have been listed in the Introduction, we close with some extensions. First, it would be interesting to check the robustness of our results when we move to an open economy and in particular to an economy which is a member of a monetary union meaning that only fiscal policy can be used for national stabilization. Second, it would be interesting to study a two-country model, where the countries differ in the degree of fiscal imbalances and so in the size of debt reduction required. Finally, it would be interesting to add agent heterogeneity, in particular, to distinguish between private and public employees. The related literature has used representative agent models so issues of income and welfare distribution have not been studied. We leave these extensions for future work.
\section*{3.8 Appendix A}

\subsection*{3.8.1 Households}

This Appendix provides details for the household’s problem. There are \( i = 1, 2, \ldots, N \) households. Each household \( i \) acts competitively to maximize expected lifetime utility.

\textbf{Household’s problem}  Household \( i \)'s expected lifetime utility is:

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, n_{i,t}, m_{i,t}, g_t) \tag{228} \]

where \( c_{i,t} \) is \( i \)'s consumption bundle (defined below), \( n_{i,t} \) is \( i \)'s hours of work, \( m_{i,t} \equiv \frac{M_{i,t}}{P_t} \) is \( i \)'s real money balances, \( g_t \) is per capita public spending, \( 0 < \beta < 1 \) is the time discount rate, and \( E_0 \) is the rational expectations operator conditional on the current period information set.

In our numerical solutions, we use the period utility function (see also e.g. Gali, 2008):

\[ u_{i,t}(c_{i,t}, n_{i,t}, m_{i,t}, g_t) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \frac{n_{i,t}^{1+\eta}}{1+\eta} + \frac{m_{i,t}^{1-\mu}}{1-\mu} + \frac{g_t^{1-\zeta}}{1-\zeta} \tag{229} \]

where \( \chi_n, \chi_m, \chi_g, \sigma, \eta, \mu, \zeta \) are preference parameters.

The period budget constraint of each household \( i \) is in nominal terms:

\[ (1 + \tau_{i,t}^c) P_t c_{i,t} + P_t x_{i,t} + B_{i,t} + M_{i,t} = (1 - \tau_{i,t}^b) (r^b_k k_{i,t-1} + D_{i,t}) + (1 - \tau_{i,t}^d) W_i n_{i,t} + R_{i,t-1} B_{i,t-1} + M_{i,t-1} - T_{i,t} \tag{230} \]

where \( P_t \) is the general price index, \( x_{i,t} \) is \( i \)'s real investment, \( B_{i,t} \) is \( i \)'s end-of-period nominal government bonds, \( M_{i,t} \) is \( i \)'s end-of-period nominal money balances, \( r^k_b \) is the real return to inherited capital, \( k_{i,t-1} \), \( D_{i,t} \) is \( i \)'s nominal dividends paid by firms, \( W_i \) is the nominal wage rate, \( R_{i,t-1} \) is the gross nominal return to government bonds between \( t-1 \) and \( t \), \( T_{i,t} \) is nominal lump-sum taxes/transfers to each \( i \) from the government, and \( \tau_{i,t}^c, \tau_{i,t}^b, \tau_{i,t}^d \) are respectively tax rates on private consumption, capital income and labour income.

Dividing by \( P_t \), the budget constraint of each \( i \) in real terms is:

\[ (1 + \tau_{i,t}^c) c_{i,t} + x_{i,t} + b_{i,t} + m_{i,t} = (1 - \tau_{i,t}^b) (r^b_k k_{i,t-1} + d_{i,t}) + (1 - \tau_{i,t}^d) w_i n_{i,t} + R_{i,t-1} \frac{P_{i,t}}{P_{i,t-1}} b_{i,t-1} + \frac{P_{i,t}}{P_{i,t-1}} m_{i,t-1} - T_{i,t} \tag{231} \]

where small letters denote real variables, i.e. \( m_{i,t} \equiv \frac{M_{i,t}}{P_t} \), \( b_{i,t} \equiv \frac{B_{i,t}}{P_t} \), \( w_t \equiv \frac{W_t}{P_t} \), \( d_{i,t} \equiv \frac{D_{i,t}}{P_t} \), \( \tau_{i,t}^f \equiv \frac{\tau_{i,t}^f}{P_t} \), at individual level.

The motion of physical capital for each household \( i \) is:

\[ k_{i,t} = (1 - \delta) k_{i,t-1} + x_{i,t} \tag{232} \]

where \( 0 < \delta < 1 \) is the depreciation rate of capital.

Household \( i \)'s consumption bundle at \( t \), \( c_{i,t} \), is a composite of \( h = 1, 2, \ldots, N \) varieties of goods, denoted as \( c_{i,t}(h) \), where each variety \( h \) is produced monopolistically by one firm \( h \). Using a Dixit-Stiglitz aggregator, we define:

\[ c_{i,t} = \left( \sum_{h=1}^{N} \lambda[c_{i,t}(h)]^{\frac{\phi_i}{\phi_i - 1}} \right)^{\frac{\phi_i - 1}{\phi_i}} \tag{233} \]
where $\phi > 0$ is the elasticity of substitution across goods produced and $\sum_{h=1}^{N} \lambda = 1$ are weights (to avoid scale effects, we assume $\lambda = 1/N$).

Household $i$’s total consumption expenditure is:

$$P_t c_{i,t} = \sum_{h=1}^{N} \lambda P_t(h) c_{i,t}(h)$$

(234)

where $P_t(h)$ is the price of variety $h$.

**Household’s optimality conditions** Each household $i$ acts competitively taking prices and policy as given. Following the literature, to solve the household’s problem, we follow a two-step procedure. Thus, we first suppose that the household chooses its desired consumption of the composite good, $c_{i,t}$, and, in turn, chooses how to distribute its purchases of individual varieties, $c_{i,t}(h)$. Details are available upon request. Then, the first-order conditions include the budget constraint above and:

$$\frac{c_{i,t}^{-\phi}}{(1 + \tau_t)} = \beta E_t \frac{c_{i,t+1}^{-\phi}}{(1 + \tau_{t+1})} \left[ (1 - \tau_{t+1}^k) r_{t+1}^k + (1 - \delta) \right]$$

(235)

$$\frac{c_{i,t}^{-\phi}}{(1 + \tau_t)} = \beta E_t \frac{c_{i,t+1}^{-\phi}}{(1 + \tau_{t+1})} R_t \frac{P_t}{P_{t+1}}$$

(236)

$$\chi_m n_{i,t}^{-\mu} = \frac{c_{i,t}^{-\phi}}{(1 + \tau_t)} + \beta E_t \frac{c_{i,t+1}^{-\phi}}{(1 + \tau_{t+1})} \frac{P_t}{P_{t+1}} = 0$$

(237)

$$\chi_n c_{i,t}^{-\mu} = \frac{(1 - \tau_{t}^n)}{(1 + \tau_t^f)} w_t$$

(238)

$$c_{i,t}(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} c_{i,t}$$

(239)

Equations (235) and (236) are respectively the Euler equations for capital and bonds, (237) is the optimality condition for money balances, (238) is the optimality condition for work hours and (239) shows the optimal demand for each variety of goods.

**Implications for price bundles** Equations (234) and (239) imply that the general price index is (see also e.g. Wickens, 2008, chapter 7):

$$P_t = \left[ \sum_{h=1}^{N} \lambda [P_t(h)]^{1-\phi} \right]^{-\frac{1}{\phi}}$$

(240)

### 3.8.2 Firms

This Appendix provides details for the firm’s problem. There are $h = 1, 2, ..., N$ firms. Each firm $h$ produces a differentiated good of variety $h$ under monopolistic competition facing Calvo-type nominal fixities. Each $i$ acts competitively to maximize expected lifetime utility.
Demand for firm’s product  Each firm $h$ faces demand for its product, $y_t(h)$, coming from households’ consumption and investment, $c_t(h)$ and $x_t(h)$, where $c_t(h) \equiv \sum_{i=1}^{N} c_{i,t}(h)$ and $x_t(h) \equiv \sum_{i=1}^{N} x_{i,t}(h)$, and from the government, $g_t(h)$. Thus, the demand for each firm’s product is:

$$y_t(h) = c_t(h) + x_t(h) + g_t(h)$$  \hfill (241)

where from above:

$$c_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} c_t$$ \hfill (242)

and similarly:

$$x_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} x_t$$ \hfill (243)

$$g_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} g_t$$ \hfill (244)

where $c_t \equiv \sum_{i=1}^{N} c_{i,t}$, $x_t \equiv \sum_{i=1}^{N} x_{i,t}$, and $g_t$ is public spending.

Since, at the economy level:

$$y_t = c_t + x_t + g_t$$ \hfill (245)

the above equations imply that the demand for each firm’s product is:

$$y_t(h) = c_t(h) + x_t(h) + g_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} y_t$$ \hfill (246)

Firm’s problem  Each firm $h$ nominal profits in period $t$, $D_t(h)$, defined as:

$$D_t(h) = P_t(h) y_t(h) - P_t r^h_k k_{t-1}(h) - W_t n_t(h)$$ \hfill (247)

All firms use the same technology represented by the production function:

$$y_t(h) = A_t [k_{t-1}(h)]^\alpha [n_t(h)]^{1-\alpha}$$ \hfill (248)

where $A_t$ is an exogenous stochastic TFP process whose motion is defined below.

Under imperfect competition, profit maximization is subject to:

$$y_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} y_t$$ \hfill (249)

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In each period, firm $h$ faces an exogenous probability $\theta$ of not being able to reset its price. A firm $h$, which is able to reset its price, chooses its price $P^h_t(h)$ to maximize the sum of discounted expected nominal profits for the next $k$ periods in which it may have to keep its price fixed.
Firm's optimality conditions  Following the related literature, to solve the firm's problem above, we follow a two-step procedure. We first solve a cost minimization problem, where each firm \( h \) minimizes its cost by choosing factor inputs given technology and prices. The solution will give a minimum nominal cost function, which is a function of factor prices and output produced by the firm. In turn, given this cost function, each firm, which is able to reset its price, solves a maximization problem by choosing its price.

The solution to the cost minimization problem gives the input demand functions:

\[
\begin{align*}
 w_t &= mc_t(1 - a)A_t[k_{t-1}(h)]^\alpha[n_t(h)]^{-\alpha} \\
 r^k_t &= mc_t a A_t[k_{t-1}(h)]^{\alpha - 1}[n_t(h)]^{1-\alpha}
\end{align*}
\]

where \( mc_t = \Psi'(\cdot) \) is the marginal nominal cost with \( t(\cdot) \) denoting the associated minimum nominal cost function for producing \( y_t(h) \) at \( t \).

Then, the firm chooses its price, \( P^\#_t(h) \), to maximize nominal profits written as:

\[
E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} D_{t+k}(h) = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P^\#_t(h) y_{t+k}(h) - \Psi_{t+k}(y_{t+k}(h)) \right\}
\]

where \( \Xi_{t,t+k} \) is a discount factor taken as given by the firm and where \( y_{t+k}(h) = \left[ \frac{P^\#_t(h)}{P_t} \right]^{-\phi} y_t \).

The first-order condition gives:

\[
E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P^\#_t(h)}{P_{t+k}} \right]^{-\phi} y_{t+k} \left\{ P^\#_t(h) - \frac{\phi}{\phi - 1} \Psi'_{t+k} \right\} = 0
\]

We transform the above equation by dividing with the aggregate price index, \( P_t \):

\[
E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P^\#_t(h)}{P_{t+k}} \right]^{-\phi} y_{t+k} \left\{ \frac{P^\#_t(h)}{P_t} - \frac{\phi}{\phi - 1} mc_{t+k} \frac{P_{t+k}}{P_t} \right\} = 0
\]

Therefore, the behaviour of each firm \( h \) is summarized by the above three conditions (250), (251) and (253).

Each firm \( h \) which can reset its price in period \( t \) solves an identical problem, so \( P^\#_t(h) = P^\#_t \) is independent of \( h \), and each firm \( h \) which cannot reset its price just sets its previous period price \( P_t(h) = P_{t-1}(h) \). Then, it can be shown that the evolution of the aggregate price level is given by:

\[
(P_t)^{1-\phi} = \theta (P_{t-1})^{1-\phi} + (1 - \theta) \left( P^\#_t \right)^{1-\phi}
\]

3.8.3 Decentralized equilibrium (given policy)

We now combine the above to solve for a Decentralized Equilibrium (DE) for any feasible monetary and fiscal policy. In this DE, (i) all households maximize utility (ii) a fraction \( (1 - \theta) \) of firms maximize profits by choosing the identical price \( P^\#_t \), while the rest, \( \theta \), set their previous period prices (iii) all constraints are satisfied and (iv) all markets clear (details are available upon request).

The DE can be summarized by the following equilibrium conditions (quantities are in per capita terms):
\[
\begin{align*}
\frac{c_t^{-\sigma}}{(1 + \tau_t^* \gamma)} &= \beta E_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^* \gamma)} \left[(1 - \tau_{t+1}^* \gamma) r_{t+1}^k + (1 - \delta)\right] \quad (255) \\
\frac{c_t^{-\sigma}}{(1 + \tau_t^* \gamma)} &= \beta E_t R_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^* \gamma)} \frac{P_t}{P_{t+1}} \quad (256) \\
x_m m_t^{e\mu} - \frac{c_t^{-\sigma}}{(1 + \tau_t^* \gamma)} - \beta E_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^* \gamma)} \frac{P_t}{P_{t+1}} &= 0 \quad (257) \\
x_n \frac{n_t^\gamma}{c_t^{-\sigma}} &= (1 - \tau_t^\gamma) w_t \quad (258) \\
k_t &= (1 - \delta) k_{t-1} + x_t \quad (259)
\end{align*}
\]

\[
E_t \sum_{k=0}^\infty \theta^k \left\{ \Xi_{t,t+k} \left[ \frac{P_t^\#}{P_{t+k}} \right]^{-\phi} y_{t+k} \left( \frac{P_t^\#}{P_t} - \phi - 1 \right) m_{t+k} \frac{P_{t+k}}{P_t} \right\} = 0 \quad (260)
\]

\[
w_t = mc_t (1 - a) \frac{y_t}{n_t} \quad (261)
\]

\[
r_t^k = mc_t a \frac{y_t}{k_t} \quad (262)
\]

\[
d_t = y_t - w_t n_t - r_t^k k_{t-1} \quad (263)
\]

\[
y_t = \frac{1}{(\frac{P_t}{P_t^\#})^{-\sigma}} A_t k_{t-1}^a n_t^{1-a} \quad (264)
\]

\[
b_t + m_t = R_{t-1} b_{t-1} \frac{P_{t-1}}{P_t} + m_{t-1} \frac{P_{t-1}}{P_t} + g_t - \tau_t^e c_t - \tau_t^g w_t n_t - \tau_t^k (r_t^k k_{t-1} + d_t) - \tau_t^l \quad (265)
\]

\[
y_t = ct + xt + gt \quad (266)
\]

\[
(P_t)^{1-\phi} = \theta (P_{t-1})^{1-\phi} + (1 - \theta) \left( P_t^\# \right)^{1-\phi} \quad (267)
\]

\[
(\tilde{P}_t)^{-\phi} = \theta (\tilde{P}_{t-1})^{-\phi} + (1 - \theta) \left( P_t^\# \right)^{-\phi} \quad (268)
\]

where $\Xi_{t,t+k} \equiv \beta^k \frac{c_t^{-\sigma}}{c_t^{-\sigma}} \frac{P_t}{P_{t+k}} \tau_t^k$ and $\tilde{P}_t^{Hi} \equiv \left( \sum_{k=1}^N [P_t (h)]^{-\phi} \right)^{-\frac{1}{\phi}}$. Thus, $\left( \frac{P_t^\#}{P_t} \right)^{-\phi}$ is a measure of price dispersion.

We thus have 14 equilibrium conditions for the DE. To solve the model, we need to specify the policy regime and thus classify policy instruments into endogenous and exogenous. Regarding the conduct of monetary policy, we assume that the nominal interest rate, $R_t$, is used as a policy instrument, while, regarding fiscal policy, we assume that the residually determined public financing policy instrument is the end-of-period public debt, $b_t$. Then, the 14 endogenous variables are $\{y_t, c_t, n_t, x_t, k_t, m_t, b_t, P_t, P_t^\#, \tilde{P}_t, w_t, mc_t, d_t, r_t^k \}_{t=0}^\infty$. This is given the independently set policy instruments, $\{R_t, \tau_t^e, \tau_t^g, \tau_t^k, \tau_t^l, g_t \}_{t=0}^\infty$, technology, $\{A_t\}_{t=0}^\infty$, and initial conditions for the state variables.
3.8.4 Decentralized equilibrium transformed (given policy)

Before we specify the motion of independently set policy instruments and exogenous stochastic variables, we rewrite the above equilibrium conditions, first, by using inflation rates rather than price levels, second, by writing the firm’s optimality condition (260) in recursive form and, third, by introducing a new equation that helps us to compute expected discounted lifetime utility. Details for each step are available upon request.

Variables expressed in ratios We define three new endogenous variables, which are the gross inflation rate \( \Pi_t \equiv \frac{P_t}{P_{t-1}} \), the auxiliary variable \( \Theta_t \equiv \frac{P^\#_t}{P_t} \), and the price dispersion index \( \Delta_t \equiv \left[ \frac{P_t}{P^\#_t} \right]^{-\phi} \). We also find it convenient to express the two exogenous fiscal spending policy instruments as ratios of GDP, \( s^g_t \equiv \frac{g_t}{y_t} \) and \( s^l_t \equiv \frac{l_t}{y_t} \).

Thus, from now on, we use \( \Pi_t, \Theta_t, \Delta_t, s^g_t, s^l_t \) instead of \( P_t, P^\#_t, \tilde{P}_t, g_t, \tau^\epsilon_t \) respectively.


\[
E_t \sum_{k=0}^{\infty} \theta^k \xi_{t+k} \left[ \frac{P^\#_{t+k}}{P_{t+k}} \right]^{-\phi} y_{t+k} \left\{ \frac{P^\#_t}{P_t} - \frac{\phi}{\phi - 1} mc_{t+k} \frac{P_{t+k}}{P_t} \right\} = 0 \tag{269}
\]

We define two auxiliary endogenous variables:

1. \( z^1_t \equiv E_t \sum_{k=0}^{\infty} \theta^k \xi_{t+k} \left[ \frac{P^\#_{t+k}}{P_{t+k}} \right]^{-\phi} y_{t+k} \frac{P^\#_t}{P_t} \) \tag{270}
2. \( z^2_t \equiv E_t \sum_{k=0}^{\infty} \theta^k \xi_{t+k} \left[ \frac{P^\#_{t+k}}{P_{t+k}} \right]^{-\phi} y_{t+k} mc_{t+k} \frac{P_{t+k}}{P_t} \) \tag{271}

Using these two auxiliary variables, \( z^1_t \) and \( z^2_t \), we come up with two new equations which enter the dynamic system and allow a recursive representation of (269). In particular, we can replace equilibrium equation (269) with:

\[
z^1_t = \frac{\phi}{(\phi - 1)} z^2_t \tag{272}
\]

where:

\[
z^1_t = \Theta_t^{-\phi - 1} y_t + \beta \theta E_t \frac{e^\epsilon_{t+1}}{c^\epsilon_{t+1}} \frac{1 + \tau^\epsilon_t}{1 + \tau^\epsilon_{t+1}} \left[ \frac{\Theta_t}{\Theta_{t+1}} \right]^{-\phi - 1} \left( \frac{1}{\Pi_{t+1}} \right)^{-\phi} z^1_{t+1} \tag{273}
\]

\[
z^2_t = \Theta_t^{-\phi} y_t mc_t + \beta \theta E_t \frac{e^\epsilon_{t+1}}{c^\epsilon_{t+1}} \frac{1 + \tau^\epsilon_t}{1 + \tau^\epsilon_{t+1}} \left[ \frac{\Theta_t}{\Theta_{t+1}} \right]^{-\phi} \left( \frac{1}{\Pi_{t+1}} \right)^{1-\phi} z^2_{t+1} \tag{274}
\]

Thus, from now on, we use (272), (273) and (274) instead of (269).
Lifetime utility written as a first-order dynamic equation. To compute social welfare, we follow Schmitt-Grohé and Uribe (2007) by defining a new endogenous variable, \( V_t \), whose motion is:

\[
V_t = \frac{c_t^{1-\sigma}}{1-\sigma} - x_n \frac{n^1_t}{1+\phi} + \chi_m \frac{m^1_t}{1-\mu} + \chi_g \frac{(s^g_t y_t)^{1-\zeta}}{1-\zeta} + \beta E_t V_{t+1}
\]  

(275)

where \( V_t \) is the expected discounted lifetime utility of the household at any \( t \).

Thus, from now on, we add equation (275) and the new variable \( V_t \) to the equilibrium system.

Equations of transformed DE. Using all the above, the final non-linear stochastic equilibrium system is:

\[
\frac{c_t^{1-\sigma}}{(1 + \tau_t^1)} = \beta E_t \frac{c_{t+1}^{1-\sigma}}{(1 + \tau_{t+1}^1)} \left[ (1 - \tau_t^k) r_t^k + (1 - \delta) \right]
\]  

(276)

\[
\frac{c_t - \sigma}{R_t} \frac{1}{(1 + \tau_t^1)} = \beta E_t \frac{c_{t+1} - \sigma}{(1 + \tau_{t+1}^1)} \frac{1}{\Pi_{t+1}}
\]  

(277)

\[
\chi_m m_t^{-\mu} - \frac{c_t^{\sigma}}{(1 + \tau_t^1)} + \beta E_t \frac{c_{t+1}^{\sigma}}{(1 + \tau_{t+1}^1)} \frac{1}{\Pi_{t+1}} = 0
\]  

(278)

\[
\frac{n^{2}_t}{c_t^{\sigma}} = \frac{(1 - \tau_t^n)}{(1 + \tau_t^1)} w_t
\]  

(279)

\[
k_t = (1 - \delta) k_{t-1} + x_t
\]  

(280)

\[
z_t^1 = \frac{\phi - 1}{\phi} \frac{z_t^2}{z_t^1}
\]  

(281)

\[
w_t = mc_t(1 - a) \frac{y_t}{n_t}
\]  

(282)

\[
r_t^k = mc_t a \frac{y_t}{k_{t-1}}
\]  

(283)

\[
d_t = y_t - w_t n_t - r_t^k k_{t-1}
\]  

(284)

\[
y_t = \frac{1}{A_t} k_t^{-a} n_t^{1-a}
\]  

(285)

\[
b_t + m_t = R_{t-1} b_{t-1} + \frac{1}{\Pi_t} + m_{t-1} + \frac{1}{\Pi_t} + s_t^2 y_t - \tau_t^e c_t - \tau_t^n w_t - \tau_t^k [r_t^k k_{t-1} + d_t] - \tau_t^l
\]  

(286)

\[
y_t = c_t + x_t + s_t^2 y_t
\]  

(287)

\[
\Pi_t^{1-\phi} = \theta (1-\theta) [\Theta_t \Pi_t]^{1-\phi}
\]  

(288)

\[
\Delta_t = (1-\theta) \Theta_t^{-\phi} + \theta \Pi_t^\phi \Delta_{t-1}
\]  

(289)

\[
z_t^1 = y_t mc_t \Theta_t^{\phi-1} + \beta \Theta_t^{\phi-1} \frac{c_{t+1}^{1-\sigma}}{c_t^{1-\sigma}} \frac{1}{1 + \tau_{t+1}^1} \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{-\phi-1} \Pi_{t+1} z_{t+1}^1
\]  

(290)
Each firm $h$ faces a quadratic price adjustment cost:

$$P_t \left( \frac{P_t(h)}{P_{t-1}(h)} - 1 \right)^2 y_t$$

Thus, each firm $h$ nominal profits, $D_t(h)$, are:

$$D_t(h) = P_t^\#(h)y_t(h) - \Psi_t(y_t(h)) - \frac{\vartheta}{2} P_t \left( \frac{P_t(h)}{P_{t-1}(h)} - 1 \right)^2 y_t$$

where $\Psi_t(y_t(h))$ is the minimum cost function.

Thus each firm $h$ maximizes the sum expected discounted nominal profits by choosing its price, $P_t(h)$:

$$\max_{P_t(h)} \sum_{t=0}^{\infty} E_t \Xi_{t,t} D_t(h)$$

$$= \sum_{t=0}^{\infty} E_t \Xi_{t,t} \left\{ P_t(h)y_t(h) - \Psi_t(y_t(h)) - \frac{\vartheta}{2} P_t \left( \frac{P_t(h)}{P_{t-1}(h)} - 1 \right)^2 y_t \right\}$$

Plugging into the objective function (295) the constraint $y_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\varphi} y_t$ we get:

$$\max_{P_t(h)} E_t \sum_{t=0}^{\infty} \Xi_{t,t} \left\{ P_t(h) \left[ \frac{P_t(h)}{P_t} \right]^{-\varphi} y_t - \Psi_t \left( \frac{P_t(h)}{P_t} \right)^{-\varphi} y_t - \vartheta P_t \left( \frac{P_t(h)}{P_{t-1}(h)} - 1 \right)^2 y_t \right\}$$

The first-order condition with respect to $P_t(h)$ is:

$$\Xi_{t,t} \left\{ (1 - \varphi) \left( \frac{P_t(h)}{P_t} \right)^{-\varphi} y_t - \Psi_t^\prime(-\varphi) \left( \frac{P_t(h)}{P_t} \right)^{-\varphi} y_t - \vartheta P_t \left( \frac{P_t(h)}{P_{t-1}(h)} - 1 \right) y_t \frac{1}{P_{t-1}(h)} \right\} +$$

$$\Xi_{t,t+1} \left\{ -\vartheta P_{t+1} \left( \frac{P_{t+1}(h)}{P_t(h)} - 1 \right) y_{t+1} \left( -\frac{P_{t+1}(h)}{[P_t(h)]^2} \right) \right\} = 0$$

Because all firms $h$ solve an identical problem we can write $P_t(h) = P_t$ and $y_t(h) = y_t$, thus:
\[
\Xi_{t,t} \left\{ (1 - \phi) y_t + \frac{\Psi'_t}{P_t^t} \phi y_t - \vartheta P_t \left( \frac{P_t}{P_t-1} - 1 \right) y_t \frac{1}{P_t-1} \right\} + \\
\Xi_{t,t+1} \left\{ -\vartheta P_{t+1} \left( \frac{P_{t+1}}{P_t} - 1 \right) y_{t+1} \left( -\frac{P_{t+1}}{P_t^2} \right) \right\} \\
= 0
\]

\[
\Xi_{t,t} \left\{ (1 - \phi) y_t + \phi mc_t y_t - \vartheta \left( \frac{P_t}{P_{t-1}} - 1 \right) y_t \frac{P_t}{P_{t-1}} \right\} + \\
\Xi_{t,t+1} \left\{ -\vartheta \left( \frac{P_{t+1}}{P_t} - 1 \right) y_{t+1} \left( -\frac{P_{t+1}}{P_t^2} \right) \right\} = 0
\] (296)

where \( \Xi_{t,t+k} = \beta^k E_t \frac{c_{i,t+k}^\sigma}{c_{i,t}^\sigma} \frac{1 + \tau_{i,t+k}^\sigma}{1 + \tau_{i,t+k}^\sigma} \frac{P_t}{P_{t+k}} \) is the nominal discount factor. Thus, equation (296) is written:

\[
(1 - \phi) y_t + \phi mc_t y_t - \vartheta \left( \frac{P_t}{P_{t-1}} - 1 \right) y_t \frac{P_t}{P_{t-1}} + \\
\beta E_t \frac{c_{i,t+k}^\sigma}{c_{i,t}^\sigma} \frac{1 + \tau_{i,t+k}^\sigma}{1 + \tau_{i,t+k}^\sigma} \frac{P_t}{P_{t+k}} \left\{ -\vartheta \left( \frac{P_{t+k}}{P_t} - 1 \right) y_{t+k} \left( -\frac{P_{t+k}}{P_t^2} \right) \right\} = 0
\]

And,

\[
(1 - \phi) y_t + \phi mc_t y_t + \beta \vartheta E_t \frac{c_{i,t+k}^\sigma}{c_{i,t}^\sigma} \frac{1 + \tau_{i,t+k}^\sigma}{1 + \tau_{i,t+k}^\sigma} \left( \frac{P_{t+k}}{P_t} - 1 \right) y_{t+k} \frac{P_{t+k}}{P_t} = \vartheta \left( \frac{P_t}{P_{t-1}} - 1 \right) y_t \frac{P_t}{P_{t-1}}
\] (297)

3.9.2 Decentralized equilibrium under Rotemberg pricing (for any feasible policy)

We now combine the above to solve for a Decentralized Equilibrium (DE) for any feasible monetary and fiscal policy. In this DE, (i) all households maximize utility (ii) all firms maximize profits facing a nominal adjustment cost by choosing an identical price \( P_t^\# \) (iii) all constraints are satisfied and (iv) all markets clear (details are available upon request).

The DE is summarized by the following equilibrium conditions (all quantities are per capita):

\[
\frac{c_{t}^\sigma}{(1 + \tau_{i,t}^\sigma)} = \beta E_t \frac{c_{t+1}^{\sigma}}{c_{t}^{\sigma}} \frac{1 + \tau_{i,t+1}^{\sigma}}{1 + \tau_{i,t+1}^{\sigma}} \left( (1 - \frac{\tau_{i,t+1}^{\sigma}}{\tau_{i,t+1}^\sigma}) \tau_{t+1}^{\sigma} + (1 - \delta) \right)
\] (298)

\[
c_{t}^{\sigma} \frac{1}{(1 + \tau_{i,t}^\sigma)} = \beta E_t R_t \frac{c_{t+1}^{\sigma}}{c_{t}^{\sigma}} \frac{P_t}{P_{t+1}}
\] (299)

\[
\chi_m m_{t}^{\mu} - \frac{c_{t}^{\sigma}}{(1 + \tau_{i,t}^\sigma)} + \beta E_t \frac{c_{t+1}^{\sigma}}{c_{t}^{\sigma}} \frac{P_t}{P_{t+1}} = 0
\] (300)

\[
\chi_n \frac{n_t^{n}}{c_{i,t}^{\sigma}} = (1 - \tau_{i,t}^{n}) w_t
\] (301)

\[
k_t = (1 - \delta) k_{t-1} + x_t
\] (302)

\[
w_t = mc_t (1 - a) \frac{\delta}{n_t}
\] (303)
\[ r_t^k = mc_t \frac{y_t}{k_t} \]  

(304)

\[ b_t + m_t = R_{t-1} b_{t-1} \frac{P_{t-1}}{P_t} + m_{t-1} \frac{P_{t-1}}{P_t} + g_t - \tau_t^c c_t - \tau_t^c w_t n_t - \tau_t^m (r_t^k k_{t-1} + d_t) - \tau_t^l \]  

(305)

\[ d_t = y_t - w_t n_t - r_t^k k_{t-1} - \frac{\phi}{2} \left( \frac{P_t}{P_{t-1}} - 1 \right)^2 y_t \]  

(306)

\[ y_t = \gamma_t^c k_{t-1} n_t^{1-a} \]  

(307)

\[ y_t = c_t + x_t + g_t + \phi \left( \frac{P_t}{P_{t-1}} - 1 \right)^2 y_t \]  

(308)

\[
(1 - \phi) y_t + \phi mc_t y_t + \beta \phi E_t \left[ \frac{\tau_{t+1}^c}{\tau_t^c} \left( \frac{P_{t+1}}{P_t} - 1 \right)^2 y_t + \frac{\tau_{t+1}^m}{\tau_t^m} (\frac{P_{t+1}}{P_t} - 1) y_t + \frac{\tau_{t+1}^l}{\tau_t^l} \right] = 0
\]  

(309)

We thus have 12 equilibrium conditions for the DE. To solve the model, we need to specify the policy regime and thus classify variables into endogenous and exogenous. Regarding the conduct of monetary policy, we assume that the nominal interest rate, \( R_t \), is used as a policy instrument, while, regarding fiscal policy, we assume that the residually determined public financing policy instrument is the end-of-period public debt, \( b_t \). Then, the 12 endogenous variables are \( \{y_t, c_t, n_t, x_t, k_t, m_t, b_t, P_t, w_t, mc_t, d_t, \tau_t^k \}_{t=0} \). This is given the independently set policy instruments, \( \{R_t, \tau_t^c, \gamma_t^c, \gamma_t^m, \gamma_t^l, A_t\}_{t=0} \), and initial conditions for the state variables.

### 3.9.3 Results with Rotemberg Pricing

Table 7 is comparable with Table 4 in subsection 3.5.1 but using Rotemberg-type instead of Calvo-type nominal fixities. The main qualitatively results remain the same, however the welfare gains from stabilization are quantitatively smaller in the case of Rotemberg-type pricing model.

<table>
<thead>
<tr>
<th>Policy instruments</th>
<th>Optimal interest-rate reaction to inflation and output</th>
<th>Optimal fiscal reaction to debt and output</th>
<th>( \xi^{16} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_t ) ( s_i^p )</td>
<td>( \phi = 3 ) ( \phi_y = 0 )</td>
<td>( \gamma_t^p = 0.1 ) ( \gamma_t^y = 0 )</td>
<td>0.0137</td>
</tr>
<tr>
<td>( R_t ) ( \tau_t^c )</td>
<td>( \phi = 3 ) ( \phi_y = 0.11 )</td>
<td>( \gamma_t^c = 0.1 ) ( \gamma_t^y = 0.63 )</td>
<td>0.0122</td>
</tr>
<tr>
<td>( R_t ) ( \tau_t^k )</td>
<td>( \phi = 3 ) ( \phi_y = 0.45 )</td>
<td>( \gamma_t^k = 3 ) ( \gamma_t^y = 3 )</td>
<td>0.0116</td>
</tr>
<tr>
<td>( R_t ) ( \tau_t^r )</td>
<td>( \phi = 3 ) ( \phi_y = 0.085 )</td>
<td>( \gamma_t^r = 0.1 ) ( \gamma_t^y = 3 )</td>
<td>0.0112</td>
</tr>
</tbody>
</table>

\( ^{16} \)The conditional welfare in the reference/passive regime is \( E_0 \hat{V}_0 = -1.9646 \).
3.10 Appendix C: How to compute optimized policy rules

3.10.1 Equilibrium conditions

The dynamic system is summarized by a set of non-linear stochastic difference equations:

\[ E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0 \]  

where \( y_t \) is a vector of \( n_y \) endogenous non-predetermined variables (controls) and \( x_t \equiv [k_t; z_t]^T \) is the vector of \( n_x \) predetermined (states) variables. The latter vector consists of two subvectors, i.e. a vector of \( n_k \) endogenous predetermined variables, \( k_t \), and a vector of \( n_z \) exogenous predetermined variables, \( z_t \). By definition \( n = n_y + n_x \) and \( n_z = n_k + n_z \). The vector of exogenous predetermined variables follows a stochastic process of the form:

\[ z_{t+1} = \tilde{h}(z_t, \sigma) + \tilde{\eta} \sigma \varepsilon_{t+1} \]

where \( \varepsilon_{t+1} \) is a vector with iid shocks with mean zero. \( \tilde{h} \) is a (linear or non-linear) function in \( z_t \) or/and \( \sigma \), \( \tilde{\eta} \) is a diagonal matrix which contains the standard deviations of the exogenous iid shocks in its main diagonal with size \( n_z \times n_z \) and \( \sigma \) is a scalar.

3.10.2 Second order accurate solution of the above system

Schmitt-Grohé and Uribe (2004) assume that the solution of the dynamic system is a function of the predetermined vector \( x_t \) and the parameter \( \sigma \) which scales the amount of uncertainty in the economy (this summarizes what they call a perturbation method). The solution is of the form:

\[ y_t = g(x_t, \sigma) \]

\[ x_{t+1} = h(x_t, \sigma) + \eta \sigma \varepsilon_{t+1} \]

where \( g \) and \( h \) are non-linear and unknown functions and the perturbation parameter, \( \sigma \), is a scalar which captures uncertainty.

The matrix \( \tilde{\eta} = \begin{bmatrix} 0_{n_z \times n_z} & \tilde{\eta} \end{bmatrix} \) contains a matrix with zeros, \( 0_{n_x \times n_x} \), in which its rows correspond to the number of the endogenous and the exogenous state variables and its columns correspond to the number of the exogenous iid shocks. While \( \tilde{\eta} \) is as defined above.

**Simple Case where the vectors \( y \) and \( x \) are scalars.** For simplicity, in this section we assume that \( y_t \) and \( x_t \) are scalars rather than vectors. The second order approximation of (312 & 313) around the non-stochastic steady-state, defined by \((x_t, \sigma) = (x, 0)\), is:

\[ \hat{y}_t = g_x(x, 0)(x_t - x) + g_\sigma(x, \sigma) \sigma + \frac{1}{2} g_{xx}(x, 0)(x_t - x)^2 + \frac{1}{2} g_{x\sigma}(x, 0)(x_t - x) \sigma + \frac{1}{2} g_{\sigma\sigma}(x, 0) \sigma^2 \]

and similarly:

\[ \hat{x}_{t+1} = h_x(x, 0)(x_t - x) + h_\sigma(x, \sigma) \sigma + \frac{1}{2} h_{xx}(x, 0)(x_t - x)^2 + \frac{1}{2} h_{x\sigma}(x, 0)(x_t - x) \sigma + \frac{1}{2} h_{\sigma\sigma}(x, 0) \sigma^2 \]
Then, we add \((322)\) to the system \((310)\), where because matrices
\[
V = \begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n
\end{bmatrix}
\]
variables containing non-predetermined vector, \(y\).
For instance, we use a within-period utility of the form:
\[
\tilde{U}_t = \sum_{t=0}^{\infty} \beta^t U_t (c_t, m_t, n_t, g_t)
\]
where variables without time subscript denote their steady-state values, \(g_x (x, 0)\) denotes the first derivative of \(g\) with respect to \(x_t\), evaluated at the steady-state with zero uncertainty, \(g_{xx} (x, 0)\) the second derivative of \(g\) with respect to \(x_t\) etc, and for the moment \(\tilde{y}_t \equiv y_t - y\). Since
\[
g_{x} = h_{x} = h_{xx} = g_{xx} = g_{x\sigma} = 0
\]
(see proof in Schmitt-Grohé and Uribe, 2004), the second order accurate solution is written:
\[
\tilde{y}_t = g_x (x, 0) (x_t - x) + \frac{1}{2} g_{xx} (x, 0) (x_t - x)^2 + \frac{1}{2} g_{x\sigma} (x, 0) \sigma^2
\]
(316)
and similarly:
\[
\tilde{x}_{t+1} = h_x (x, 0) (x_t - x) + \frac{1}{2} h_{xx} (x, 0) (x_t - x)^2 + \frac{1}{2} h_{x\sigma} (x, 0) \sigma^2
\]
(317)
In Schmitt-Grohé and Uribe (2004) they provide numerical routines which find the matrices of the solution \(g_x, h_x, g_{xx}, y_{x\sigma}, h_{xx}, h_{x\sigma}\).

The case where \(y\) and \(x\) are vectors In this case the notation is much trickier. The second order accurate solution is written:
\[
[\tilde{x}_{t+1}]_j = [h_x]_j [\tilde{x}_t]_a + \frac{1}{2} \left\{ [h_{xx}]_{ab} [\tilde{x}_t]_a [\tilde{x}_t]_b + [h_{x\sigma}]_i \sigma^2 \right\} + [\eta]_c \sigma [\tilde{e}_{t+1}]_c
\]
(318)
\[
[\tilde{y}_t]_j = [g_x]_j [\tilde{x}_t]_a + \frac{1}{2} \left\{ [g_{xx}]_{ab} [\tilde{x}_t]_a [\tilde{x}_t]_b + [g_{x\sigma}]_i \sigma^2 \right\}
\]
(319)
where \(j = 1, 2, ..., n_x\), \(a, b = 1, 2, ..., n_x\), \(c = 1, 2, ..., n_z\) and \(i = 1, 2, ..., n_y\). This notation is used because matrices \([h_{xx}]_{ab}\) and \([g_{xx}]_{ab}\) have three dimensions. Also, \(\tilde{x}_t \equiv x_t - x\) and \(\tilde{y}_t \equiv y_t - y\), variables without time subscript denote steady-state values.

3.10.3 Welfare analysis
In this section, we analyze how we use the second order accurate solution found above so as to make welfare analysis. Particularly, we introduce into the model a welfare measure, i.e. life-time utility of households.

3.10.4 Adding a new dynamic equation for life-time utility
We now follow Schmitt-Grohé and Uribe (2007). In our model, the life-time utility (welfare) is defined as:
\[
V_t \equiv E_t \sum_{t=0}^{\infty} \beta^t U_t (c_t, m_t, n_t, g_t)
\]
(320)
where \(c_t\) is consumption, \(m_t\) is real money balances, \(n_t\) hours of work and \(g_t\) government spending per capita. For instance, we use a within-period utility of the form:
\[
u_t = \log (c_t) - \chi_n \frac{(n_t)^{1+\eta}}{1+\eta} + \chi_m \frac{(m_t)^{1-\mu}}{1-\mu} + \chi_g \log (g_t)
\]
(321)
Following Schmitt-Grohé and Uribe (2007), we rewrite (320) as a dynamic equation:
\[
V_t = \log (c_t) - \chi_n \frac{(n_t)^{1+\eta}}{1+\eta} + \chi_m \frac{(m_t)^{1-\mu}}{1-\mu} + \chi_g \log (g_t) + \beta E_t V_{t+1}
\]
(322)
Then, we add (322) to the system (310), where \(V_t\) is a new control variable included in the non-predetermined vector, \(y_t\). Therefore, the first element of the vector of non-predetermined variables contains \(V_t\), i.e. \(y_t (1)\).
3.10.5 Add the feedback policy rules as dynamic equations

Similarly, we assume that the independently set policy instruments, \( R_t, s_t, \tau_t^e, \tau_t^k, \tau_t^n \), follow feedback policy rules:

\[
\log \left( \frac{R_t}{R_t} \right) = \phi_x \log \left( \frac{R_t}{R_t} \right) + \phi_y \log \left( \frac{y_t}{y} \right) \tag{323}
\]

\[
s_t^g - s^g = -\gamma_t^g (t_{t-1} - I) - \gamma_t^g (y_t - y) \tag{324}
\]

\[
\tau_t^c - \tau^c = \gamma_t^c (t_{t-1} - I) + \gamma_t^c (y_t - y) \tag{325}
\]

\[
\tau_t^k - \tau^k = \gamma_t^k (t_{t-1} - I) + \gamma_t^k (y_t - y) \tag{326}
\]

\[
\tau_t^n - \tau^n = \gamma_t^n (t_{t-1} - I) + \gamma_t^n (y_t - y) \tag{327}
\]

Thus, we augment the dynamic system (310) by 5 new equations (323-327) and the vector of endogenous variables with 5 new endogenous variables \( R_t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n \).

3.10.6 Determinacy of DE

So far, we have added welfare and the independently set policy instruments as 6 new endogenous variables in the model by augmenting the dynamic system with 6 new equilibrium equations. The existence of a unique first order accurate solution of the model depends crucially on the values of the ten feedback policy coefficients, i.e. \( \phi_x, \phi_y, \gamma_t^c, \gamma_t^k, \gamma_t^e, \gamma_t^o, \gamma_t^p, \gamma_t^y, \gamma_t^g \). So before we proceed with welfare analysis we find the ranges of the feedback policy coefficients for which we get determinacy, in other words a unique stable equilibrium. In what follows we guarantee that we are always in the determinate areas of the model.

3.10.7 Welfare as a function of the state variables and the exogenous shock

Now, the dynamic system (310) contains also equations (322) and (323-327). Consequentially, we can obtain a solution for life-time utility, \( V_t \), as a function of the vector of states and the perturbation parameter, \( \sigma \):

\[
V_t = g^V (x_t, \sigma) \tag{328}
\]

However, in practice \( g^V (\cdot) \) is an unknown function but using Schmitt-Grohé and Uribe (2004) we take a second order accurate solution of it. Remember that \( V_t \) is intentionally the first element of the vector of the endogenous variables so the second order accurate solution of it is:

\[
\hat{V}_t (1) = \hat{V}_t = [g_{x_1}^1]_{1a} \left[ \hat{x}_1 \right]_a + \frac{1}{2} \left\{ [g_{xx}]_{ab}^{1} \left[ \hat{x}_1 \right]_a \left[ \hat{x}_1 \right]_b + [g_{x\sigma}]^1 \sigma^2 \right\} \tag{329}
\]

Recall that we have also included in the dynamic system equations (323-327) which means that matrices \([g_{x_1}^1], [g_{xx}]_{ab}^{1}\) and \([g_{x\sigma}]^1\) are also functions of the ten feedback policy coefficients \( \phi_x, \phi_y, \gamma_t^c, \gamma_t^k, \gamma_t^e, \gamma_t^o, \gamma_t^p, \gamma_t^y, \gamma_t^g \) and the steady-state solution of the model. In general, different values of the feedback policy coefficients imply different solutions for life-time utility, \( \hat{V}_t \).
3.10.8 Express the unconditional expectation of welfare as a function of the feedback policy coefficients

Now, we can use the solution (329) to find the unconditional expectation of life-time utility, i.e. $E_t\hat{V}_t$. The unconditional expectation of welfare depends on the matrices of the solution (the matrices in equations 318 & 319) and as a result on the ten feedback policy coefficients too. Let us define a vector $\mathcal{F} = \{\phi_x, \phi_y, \gamma_{x}, \gamma_{y}, \gamma_{x}^{l}, \gamma_{y}^{l}, \gamma_{x}^{b}, \gamma_{y}^{b}, \gamma_{x}^{g}, \gamma_{y}^{g}\}$ which contains the feedback policy coefficients, then we can write the unconditional expectation as a function of the vector of feedbacks $\mathcal{F}$:

$$E_t\hat{V}_t = W(\mathcal{F}) \quad (330)$$

Notice that the function $W(\cdot)$ summarizes the Decentralized Equilibrium of the model, the specification of the period-by-period utility, the steady-state solution and the structure of the feedback policy coefficients. Given these, we can assign a particular value for each element of the vector of the feedback policy coefficients, $\mathcal{F}$, and find the unconditional expectation of life-time utility. In practice, we choose the vector $\mathcal{F}$ in order to maximize (330).

3.10.9 Express the conditional welfare as a function of the feedback policy coefficients and the initial values of the state variables

We can write the solution for welfare (329) as a function of the initial value of the state vector, i.e. $x_0$:

$$\hat{V}_0 = [g_{x}]_a^1 [x_0 - x]_a + \frac{1}{2} \left\{ [g_{xx}]_{ab} [x_0 - x]_a [x_0 - x]_b + [g_{x\sigma}]_a^1 \sigma^2 \right\} \quad (331)$$

It is clear from (331) that the solution of welfare depends on the initial conditions of the state variables, $x_0$. Different, initial conditions imply different conditional life-time utility $\hat{V}_0$. Say, we set the initial condition of the state vector equal to its non-stochastic steady-state value $x_0 = x$. Then, (331) is written:

$$\hat{V}_0 = \frac{1}{2} [g_{x\sigma}]_a^1 \sigma^2 \quad (332)$$
4 Optimized fiscal policy rules in a semi-small open economy with sovereign premia and without monetary policy independence

4.1 Introduction

Since the global crisis in 2008, public finances have been at the center of attention in most eurozone periphery countries. Although several policy proposals are under discussion, a particularly debated reform has been debt consolidation. Proponents claim this is for good reason: as a result of high and rising public debt, borrowing costs have increased causing financing problems and undermining government solvency. Opponents, on the other hand, claim that debt consolidation worsens the economic downturn and leads to a vicious cycle. At the same time, as members of the single currency, these countries cannot use monetary and exchange rate policy to counter the recession and/or monetize public debt. Thus, the only macroeconomic tool available is fiscal policy.

What is the best use of fiscal policy under these circumstances? Is debt consolidation productive? Since it is recognized that there is no such a thing like “the” debt consolidation, do the answers to these questions depend on which tax-spending policy instrument is used for debt consolidation? Also, since, in practice, policymakers follow simple rules according to which policy instruments react to some economic indicators, the so-called operating targets, which indicators the tax-spending policy instruments should react to, and how strong this reaction should be?

This paper ranks various fiscal policies in light of the above. The setup is the standard New Keynesian model extended into a small open-economy setting with sovereign premia. These premia mean that the interest rate, at which the country borrows from the world capital market, increases with government’s total debt (for empirical evidence, see e.g. Obstfeld and Taylor, 2003, and European Commission, 2011; for a review of the theoretical literature, see e.g. Schmitt-Grohé and Uribe, 2003). We focus on a monetary policy regime in which the semi-small open economy fixes the exchange rate and, at the same time, loses monetary policy independence; this mimics membership in a currency union. Hence, the only macroeconomic tool left is fiscal policy. We then allow public spending and the main types of tax rates to respond to the gap between actual public debt and target public debt as shares of output, as well as to the gap between actual and target output. We experiment with various targets depending on whether policymakers aim just to stabilize the economy around its status quo, or whether they also want to move the economy to a reformed long run (for instance, a new long run without sovereign premia).

The model is solved numerically using fiscal and public finance data from the Italian economy during 2001-2011. We choose Italy because it exhibits most of the features discussed in the opening paragraph above. It thus looks as a natural choice to quantify our model.

To rank different policies, and since we do not want our results to be driven by ad hoc differences in feedback policy coefficients across different policy rules, we compute optimized policy rules when the welfare criterion is household’s expected lifetime utility. In particular, we adopt the methodology of Schmitt-Grohé and Uribe (2004 and 2007), in the sense that we compute the welfare-maximizing values of feedback policy coefficients by taking a second-order approximation to both the equilibrium conditions and the welfare criterion. This enables us to welfare rank different policy rules in a stochastic setup.

Our main results are as follows. First, as expected, there is no such a thing like "the" debt consolidation. The choice of the fiscal policy instrument matters for welfare and output. This holds even if we compare optimized policy rules. The choice of the fiscal policy instrument also matters for how quickly debt should be brought down. For instance, in our experiments, public
debt reduction and elimination of sovereign premia should be achieved within a range of six to twelve years (six, if we use consumption taxes, and twelve, if we use labor taxes).

Second, irrespectively of the fiscal policy instrument used, when we compare outcomes with, and without, debt consolidation, consolidation is welfare-improving only if we are relatively far-sighted. For instance, when we use labor taxes, debt consolidation is welfare-improving if we care beyond the first fourteen years, and output-improving if we care beyond the first five years. In other words, debt consolidation always comes at a short term pain. Thus, the argument for, or against, debt consolidation involves a value judgment.

Third, contrary to policy reports\(^{17}\) and related closed-economy studies,\(^{18}\) which suggest that spending-based consolidations are better than tax-based ones, this is not the case in a semi-small open economy with sovereign premia. In particular, when we are restricted to use a single fiscal instrument at a time (meaning that we use the same instrument in the transition and in the long run), labor and, in turn, capital taxes score better than consumption taxes and public spending. This holds across all time horizons. It also continues to hold even when we are allowed to use fiscal policy mixes (meaning that the policy instrument used in the transition can be different from that used in the long run). In particular, when we compare different policy mixes, and if we care about the short- and medium-run, the best mix is to raise the labor tax rate during the early consolidation phase and to reduce the capital tax rate once the public debt burden has been reduced and fiscal space has been created.

Fourth, to understand these results, one should recall that, in general, the implications of debt consolidation depend both on which policy instrument bears the cost of adjustment in the early phase of adjustment and on which policy instrument is expected to reap the benefit, once the debt burden has been reduced and fiscal space has been created. An optimal policy should find a balance between these two. In our model, labor and capital taxes are more suitable to address the extra distortions/wedges generated by sovereign premia during the early costly period. In turn, once debt and premia have been offset and fiscal space has been created, the most efficient way to take advantage of this fiscal space is to reduce labor and capital taxes, which are now particularly distorting. The combination of these two effects (one in the early phase of fiscal pain and the other in the later phase of fiscal gain) makes labor and capital taxes more desirable than public spending and consumption taxes in a small open economy with premia.

Fifth, fiscal policy instruments should react to both public debt imbalances and the output gap. Thus, fiscal reaction to the output gap is productive. Nevertheless, if there is need to reduce public debt and sovereign premia over time (and, as said, this is welfare-improving when we far-sighted enough), then the net changes in fiscal instruments should be dominated by reaction to debt rather than to output imbalances.

This paper is related to three strands of the literature. First, it is related to the literature on how monetary and fiscal policy instruments react, or should react, to the business cycle.\(^{19}\) Second, it is related to the literature on fiscal consolidation that usually compares spending cuts versus tax rises needed for debt reduction.\(^{20}\) Third, it is related to the literature on sovereign premia.\(^{21}\) Nevertheless, as far as we know, there have not been any previous attempts to wel-

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\(^{17}\)See e.g. European Commission (2011).
\(^{18}\)See e.g. Philippopoulos et al. (2013) for a companion closed-economy model calibrated to the euro area.
\(^{20}\)See e.g. Forni et al. (2010), Bi et al. (2012), Cogan et al. (2013), Erceg and Linde (2012, 2013), Papageorgiou (2012) and Philippopoulos et al. (2013).
\(^{21}\)See e.g. Schmitt-Grohé and Uribe (2003), Bi (2012), Bi et al. (2012), Ghosh et al. (2013) and Corsetti et al. (2013). See below for further details.
fare rank the main tax-spending instruments in a New Keynesian open economy with sovereign premia and without monetary policy independence, and study how results depend on whether the government simply stabilizes the economy from shocks, or also reduces public debt and eliminates premia over time. Also, to rank alternative policies, we work with optimized feedback policy rules.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents the data, calibration and the status quo solution. Section 4 explains how we model policy. Policy experiments are in Section 5. Section 6 closes the paper. An Appendix includes details.

4.2 Model

Consider a model of a small open economy, which is extended to include endogenous sovereign premia and a rich menu of state-contingent policy rules. The rest of the setup is the standard New Keynesian model of a small open economy with domestic and imported goods featuring imperfect competition and nominal Calvo-type rigidities.\footnote{For the New Keynesian model, see e.g. the textbooks of Gali (2008) and Wickens (2008). For small open economy New Keynesian models, see e.g. Gali and Monacelli (2005, 2008).} Exogenous fluctuations are driven by temporary shocks.

The domestic economy is composed of $N$ identical households indexed by $i = 1, 2, ..., N$, of $N$ firms indexed by $h = 1, 2, ..., N$, each one of them producing a differentiated domestically produced tradable good, as well as of monetary and fiscal authorities. Similarly, there are $f = 1, 2, ..., N$ differentiated imported goods produced abroad. Population, $N$, is constant over time.

4.2.1 Aggregation and prices

Consumption bundles The quantity of variety $h$ produced at home by firm $h$ and consumed by domestic household $i$ is denoted as $c_{i,t}^H(h)$. Using a Dixit-Stiglitz aggregator, the composite of domestic goods consumed by household $i$, $c_{i,t}^H$, is given by:\footnote{As in e.g. Blanchard and Giavazzi (2003), we find it more convenient to work with summations rather than with integrals.}

\begin{equation}
    c_{i,t}^H = \left[ \frac{\sum_{h=1}^{N} \lambda [c_{i,t}^H(h)]^{\frac{\phi-1}{\phi}}}{\lambda^{\frac{\phi}{\phi-1}}} \right]^{\frac{\phi}{\phi-1}}
\end{equation}

where $\phi > 0$ is the elasticity of substitution across goods produced in the domestic country and $\lambda = 1/N$ is a weight chosen to avoid scale effects.

Similarly, the quantity of imported variety $f$ produced abroad by firm $f$ and consumed by domestic household $i$ is denoted as $c_{i,t}^F(f)$. Using a Dixit-Stiglitz aggregator, the composite of imported goods consumed by household $i$, $c_{i,t}^F$, is given by:

\begin{equation}
    c_{i,t}^F = \left[ \frac{\sum_{f=1}^{N} \lambda [c_{i,t}^F(f)]^{\frac{\phi-1}{\phi}}}{\lambda^{\frac{\phi}{\phi-1}}} \right]^{\frac{\phi}{\phi-1}}
\end{equation}

Household $i$'s consumption bundle, $c_{i,t}$, consists of domestic and foreign goods as defined by:

\begin{equation}
    c_{i,t} = \left( c_{i,t}^H \right)^{\nu} \left( c_{i,t}^F \right)^{1-\nu}
\end{equation}

where $\nu$ is the degree of preference for domestic goods (if $\nu > 1/2$, there is a home bias).
Consumption expenditure, prices and terms of trade

Household $i$’s total consumption expenditure is:

$$P_t c_{i,t} = P_t^H c_{i,t}^H + P_t^F c_{i,t}^F$$

(336)

where $P_t$ is the consumer price index (CPI), $P_t^H$ is the price index of home tradables, and $P_t^F$ is the price index of foreign tradables (expressed in domestic currency).

Each household’s total expenditure on home goods and foreign goods are respectively:

$$P_t^H c_{i,t}^H = \sum_{h=1}^{N} \lambda P_t^H(h) c_{i,t}^H(h)$$

(337)

$$P_t^F c_{i,t}^F = \sum_{f=1}^{N} \lambda P_t^F(f) c_{i,t}^F(f)$$

(338)

where $P_t^H(h)$ is the price of variety $h$ produced at home and $P_t^F(f)$ is the price of variety $f$ produced abroad, both denominated in domestic currency.

We assume that the law of one price holds meaning that each tradable good sells at the same price at home and abroad. Thus, $P_t^F(f) = S_t P_t^{H*}(f)$, where $S_t$ is the nominal exchange rate (where an increase in $S_t$ implies a depreciation) and $P_t^{H*}(f)$ is the price of variety $f$ produced abroad denominated in foreign currency. A star denotes the counterpart of a variable or a parameter in the rest-of-the-world. Note that the terms of trade are defined as $P_t^{H*}(f) = \frac{S_t^{H*}}{P_t^F}$, while the real exchange rate is defined as $\frac{S_t^{H*}}{P_t}$.

4.2.2 Households

Each household $i$ acts competitively to maximize expected lifetime utility given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, n_{i,t}, m_{i,t}, g_t)$$

(339)

where $c_{i,t}$ is $i$’s consumption bundle as defined above, $n_{i,t}$ is $i$’s hours of work, $m_{i,t}$ is $i$’s real money holdings, $g_t$ is per capita public spending, $0 < \beta < 1$ is the time discount rate, and $E_0$ is the rational expectations operator conditional on the information set.

The period utility function is assumed to be of the form (see also e.g. Gali, 2008):

$$u_{i,t}(c_{i,t}, n_{i,t}, m_{i,t}, g_t) = \frac{1-\sigma}{1-\eta} c_{i,t}^{1-\eta} - \frac{\mu}{1-\eta} m_{i,t}^{1-\mu} + \frac{1-\zeta}{1-\eta} g_{t+1}$$

(340)

where $\chi_n$, $\chi_m$, $\chi_g$, $\sigma$, $\eta$, $\mu$, $\zeta$ are preference parameters. Thus, $\sigma$ is a coefficient of intertemporal substitution and $\eta$ is the inverse of Frisch labour elasticity.

The period budget constraint of each household $i$ expressed in real terms is:

$$\left(1 + \tau^i_t\right) \left[ \frac{P_t^H H_{i,t}^H + P_t^F F_{i,t}^F}{P_t^H} + \frac{P_t^H}{P_t} x_{i,t} + b_{i,t} + m_{i,t} + \frac{S_t P^*_t}{P_t} f_{i,t} + \frac{S_t}{2} \left( \frac{S_t P^*_t}{P_t} f_{i,t} - \frac{S^* P^*_t}{P^*} f^*_t \right) \right]^2$$

$$= \left(1 - \tau^i_t\right) \left[ r^i_t \left\{ \left( 1 + \tau^i_t\right) \left[ \frac{P_t^H H_{i,t-1} + P_t^F F_{i,t-1}}{P_t^H} \right] + \frac{P_t^H}{P_t} x_{i,t-1} + b_{i,t-1} + m_{i,t-1} + \frac{S_t P^*_t}{P_t} f_{i,t-1} + \frac{S_t}{2} \left( \frac{S_t P^*_t}{P_t} f_{i,t-1} - \frac{S^* P^*_t}{P^*} f^*_t \right) \right] \right]$$

$$+ \frac{P_t^F}{P_t} c_{i,t-1} + \frac{S_t^*}{P_t^*} f_{i,t-1}^* - \tau^i_t$$

(341)

24For the relation between terms of trade and the real exchange rate, see Benigno and Thoenissen (2003). See also Fahr and Smets (2010) and Wickens (2008, p. 166).
where \(x_{i,t}\) is \(i\)'s domestic investment, \(b_{i,t}\) is \(i\)'s end-of-period real domestic government bonds, \(m_{i,t}\) is \(i\)'s end-of-period real domestic money holdings, \(f_{h,t}\) is \(i\)'s end-of-period real internationally traded assets denominated in foreign currency, \(r^k\) is the real return to inherited domestic capital, \(d_{i,t}\) is \(i\)'s real dividends received by domestic firms, \(w_t\) is the real wage rate, \(R^1\) is the gross nominal return to domestic government bonds between \(t\) and \(t+1\), \(Q^1\) is the gross nominal return to international assets between \(t\) and \(t+1\), \(l_{i,t}\) are real lump-sum taxes/transfers to each household, and \(c_t, k_t, n_t\) are tax rates on consumption, capital income and labour income respectively. Thus, small letters denote real variables, namely, \(m_{i,t}^t, b_{i,t}^t, f_{h,t}^t, w_t^t, d_{i,t}^t, \tau^t, \tau^t, \tau^n\) are nominal variables. The parameter \(\phi^h \geq 0\) measures transaction costs related to foreign assets, where variables without time subscripts denote long-run values (such costs are not important to the main results but help the model with the data - see also below).

The law of motion of physical capital for household \(i\) is:

\[
k_{i,t} = (1 - \delta)k_{i,t-1} + x_{i,t} - \frac{\xi}{2} \left( \frac{k_{i,t}}{k_{i,t-1}} - 1 \right)^2 k_{i,t-1}
\]

(342)

where \(0 < \delta < 1\) is the depreciation rate of capital and \(\xi \geq 0\) is a parameter capturing adjustment costs related to physical capital.

Each household \(i\) acts competitively taking prices and policy as given. The first-order conditions are presented in Appendix.

### 4.2.3 Implications for price bundles

The three price indexes are (see Appendix for details):

\[
P_t = (P_t^H)^\nu (P_t^F)^{1-\nu}
\]

(343)

\[
P_t^H = \left[ \sum_{h=1}^{N} \lambda_h [P_t^H(h)]^{1-\phi} \right]^{1/\phi}
\]

(344)

\[
P_t^F = \left[ \sum_{f=1}^{N} \lambda_f [P_t^F(f)]^{1-\phi} \right]^{1/\phi}
\]

(345)

### 4.2.4 Firms

Each domestic firm \(h\) produces a differentiated good of variety \(h\) under monopolistic competition facing Calvo-type nominal fixities.

Nominal profits of firm \(h\) are defined as:

\[
D_t(h) \equiv P_t^H(h) y_t^H(h) - r_t^k P_t^H(h) k_{t-1} - W_t n_t(h)
\]

(346)

All firms use the same technology represented by the production function:

\[
y_t^H(h) = A_t[k_{t-1}^{\alpha} n_t(h)]^{1-\alpha}
\]

(347)

where \(A_t\) is an exogenous stochastic TFP process whose motion is defined below.

Profit maximization by firm \(h\) is subject to the demand for its product (see Appendix for details):
that is, demand for firm $h$’s product, $y^H_t(h)$, comes from domestic households’ consumption and investment, $c^H_t(h)$ and $x_t(h)$, where $c^H_t(h) \equiv \sum_{i=1}^{N} c^H_{i,t}(h)$ and $x_t(h) \equiv \sum_{i=1}^{N} x_{i,t}(h)$, from the domestic government, $g_t(h)$, and from foreign households’ consumption, $c^F_t(h) \equiv \sum_{i=1}^{N} c^F_{i,t}(h)$.

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In particular, in each period, each firm $h$ faces an exogenous probability $\theta$ of not being able to reset its price. A firm $h$, which is able to reset its price at time $t$, chooses its price $P_t^h(h)$ to maximize the sum of discounted expected nominal profits for the next $k$ periods in which it may have to keep its price fixed. This objective is given by:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} D_{t+k} (h) = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P_t^h (h) y_t^{H_k} (h) - \Psi_{t+k} \left( y_t^{H_k} (h) \right) \right\}$$

where $\Xi_{t,t+k}$ is a discount factor taken as given by the firm, $y_t^{H_k} (h) = \left[ \frac{P_t^h(h)}{P_{t+k}^h} \right]^\phi y_t^{H_k}$ and $\Psi_t(.)$ is the minimum nominal cost function for producing $y_t^H (h)$ at $t$ so that $\Psi_t(.)$ is the associated nominal marginal cost. Details for the firm’s problem and its first-order conditions are in Appendix.

### 4.2.5 Government budget constraint

The period budget constraint of the government expressed in real terms is (in aggregate quantities):

$$b_t + m_t + \frac{S^H_t}{P_t} f^g_t = \frac{\phi^g}{2} \left( \frac{S^H_t}{P_t} f^g_t - \frac{S^F_t}{P_t} f^g_t \right)^2 + R_{t-1} \frac{P_{t-1}}{P_t} b_{t-1} + \frac{P_{t-1}}{P_t} m_{t-1} +$$

$$+ Q_{t-1} \frac{S^H_t}{P_t} \frac{P_{t-1}}{P_t} f^g_{t-1} + \frac{P^H_t}{P_t} g_t - \tau^i_t \left( \frac{P_t^H}{P_t} c^H_t + \frac{P_t^F}{P_t} c^F_t \right) - \tau^s_t \left( \frac{S_t^H}{P_t} k_{t-1} + d_t \right) - \tau^s_t w_t n_t - \tau^l_t$$

where $b_t$ is the end-of-period total domestic real public debt, $m_t$ is the end-of-period total stock of real money balances and $f^g_t$ is the end-of-period total external real public debt expressed in foreign prices. We also use $c^H_t \equiv \sum_{i=1}^{N} c^H_{i,t}$, $c^F_t \equiv \sum_{i=1}^{N} c^F_{i,t}$, $k_{t-1} \equiv \sum_{i=1}^{N} k_{i,t-1}$, $D_i \equiv \sum_{i=1}^{N} D_{i,t}$, $n_t \equiv \sum_{i=1}^{N} n_{i,t}$, $F^h_{t-1} \equiv \sum_{i=1}^{N} F^h_{i,t-1}$, $B_{t-1} \equiv \sum_{i=1}^{N} B_{i,t-1}$ and $T^i_t \equiv \sum_{i=1}^{N} T^i_{i,t}$. Notice that, as above, small letters denote real variables, namely, $b_t \equiv \frac{b_t}{P_t}$, $m_t \equiv \frac{m_t}{P_t}$, $f^g_t \equiv \frac{f^g_t}{P_t}$, $d_t \equiv \frac{d_t}{P_t}$, $w_t \equiv \frac{w_t}{P_t}$ and $\tau^i_t \equiv \frac{\tau^i_t}{P_t}$. Also, recall that the government allocates its total expenditure among product varieties $h$ by solving an identical problem with household $i$, so that $g_t (h) = \left[ \frac{P^t(h)}{P^t} \right]^\phi g_t$. The parameter $\phi^g \geq 0$ measures transaction costs similar to those of the household.

In each period, one of the fiscal policy instruments, $\tau^f_t$, $\tau^i_t$, $\tau^s_t$, $g_t$, $\tau^l_t$, $b_t$, $f^g_t$, has to follow residually to satisfy the government budget constraint (see below).

### 4.2.6 Closing the model: the world interest rate

As is known, to get a well-defined solution, we have to depart from the benchmark small open economy model. Here, following e.g. Schmitt-Grohé and Uribe (2003), we do so by endogenizing
the interest rate faced by the domestic country when it borrows from the world capital market, $Q_t$. In particular, we assume that $Q_t$ is an increasing function of total public debt as a share of output. This rather common assumption is supported by a number of empirical studies (see e.g. European Commission, 2011).\(^{25}\)

In particular, following e.g. Schmitt-Grohé and Uribe (2003) and Christiano et al. (2010), we employ the functional form:

$$Q_t = Q^*_t + \psi \left( e^{(\frac{b + s_t r_t^d}{r^d v_t^d t} - d)} - 1 \right)$$

(351)

where $Q^*_t$ is exogenously given, $d$ is an exogenous threshold value above which the interest rate on government debt starts rising above $Q^*_t$ (see below) and the parameter $\psi$ measures the elasticity of the interest rate with respect to deviations of total public debt from its threshold value.

### 4.2.7 Monetary and fiscal policy regimes

Before we solve the model, we need to specify the exchange rate and the fiscal policy regimes. Concerning the exchange rate regime, since the model is applied to Italy since 2001, we solve it for a case without monetary policy independence. In particular, we assume that the nominal exchange rate, $S_t$, is exogenously set (say, at one) and, at the same time, we choose the domestic nominal interest rate on domestic government bonds, $R_t$, to be an endogenous variable.\(^{26}\) To understand what this implies, it is better to think in terms of certainty. Then, as can be seen from the arbitrage interest rate condition, $R_t$ is determined by the world interest rate, $Q_t$, which is exogenously given to the country, and the country’s risk premium, which depends on endogenous variables.\(^{27}\) In other words, under fixed exchange rates, a semi-small country loses monetary policy independence. We believe this is a reasonable description of a small economy participating in the euro system. Concerning fiscal policy, we start by assuming that, along the transition, the residually determined public financing policy instrument is the end-of-period foreign public debt, $F_t^g$ (see below for other cases).

### 4.2.8 Decentralized equilibrium (given policy)

We now combine the above to present the Decentralized Equilibrium (DE) for any feasible policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) households maximize utility; (ii) a fraction $(1 - \theta)$ of firms maximize profits by choosing an identical price

\(^{25}\) Alternatively, to model sovereign premia, we could appeal to the notion of a fiscal limit (see e.g. Bi, 2012, Bi et al., 2012, Ghosh et al., 2013, and Corsetti et al., 2013) or to the notion that default is a strategic choice of the sovereign (see e.g. Arellano, 2008). For further details, see e.g. Corsetti et al. (2013).

\(^{26}\) This is similar to modeling in e.g. Erceg and Linde (2012). As we explain in more detail in Appendix 3, the fact that $S_t$ is exogenous does not necessarily imply that $R_t$ should become an endogenous variable. Some other policy instrument could play the role of the adjusting policy instrument, so that both $S_t$ and $R_t$ could be treated as policy instruments. We have experimented with several candidates in our numerical solutions below and we can report that, only when we treat $R_t$ as endogenous, the model gives well-defined solutions under fixed exchange rates. Recall that in the popular case of flexible, or managed, floating exchange rates, $S_t$ and $R_t$ switch positions, in the sense that $S_t$ becomes an endogenous variable, while $R_t$ is used as a policy instrument usually assumed to follow a Taylor-type rule. For the modeling of different exchange rate regimes in similar models, see e.g. Gali and Monacelli (2005), Dellas and Tavlas (2005) and Collard and Dellas (2006).

\(^{27}\) To see this, combine equations (363) and (364) in the Appendix under certainty. They imply $R_t = Q_t \frac{S_t + 1}{S_t}$, which is the uncovered interest parity condition. Under fixed exchange rates, we simply have $R_t = Q_t$, where $Q_t$ consists of the exogenous rest-of-the-world interest rate and the endogenous sovereign premium. See also e.g. Himmels and Kirsanova (2009) for a similar specification.
variables have been defined above, except from of-the-world variables, independently set monetary and fiscal policy instruments, (we report that our main results do not change when we add more indicators). In particular, that the fiscal authorities react to a small number of easily observable macroeconomic indicators (following Schmitt-Grohé and Uribe (2007) and many others, we focus on simple rules meaning without room for monetary policy independence, only fiscal policy can be used for stabilization.

4.2.9 Fiscal policy rules

Without room for monetary policy independence, only fiscal policy can be used for stabilization. Following Schmitt-Grohé and Uribe (2007) and many others, we focus on simple rules meaning (for similar rules, see e.g Schmitt-Grohé and Uribe (2007), Bi (2010) and Cantore et al. (2012)): 

\[ s_t^q - s^0 = -\gamma_t^q \left( l_{t-1} - l \right) - \gamma_t^y \left( y_t^H - y^H \right) \] (352)

\[ \tau_t^c - \tau^c = \gamma_t^c \left( l_{t-1} - l \right) + \gamma_t^y \left( y_t^H - y^H \right) \] (353)

\[ \tau_t^k - \tau^k = \gamma_t^k \left( l_{t-1} - l \right) + \gamma_t^y \left( y_t^H - y^H \right) \] (354)

\[ \tau_t^n - \tau^n = \gamma_t^n \left( l_{t-1} - l \right) + \gamma_t^y \left( y_t^H - y^H \right) \] (355)

where variables without time subscripts denote target values (defined below), \( q \equiv (g, c, k, n) \) and \( \gamma_t^q \geq 0 \) and \( \gamma_t^n \geq 0 \) are feedback fiscal policy coefficients on the output share of inherited public liabilities, \( l_{t-1} \), and output, \( y_t^H \), respectively, as deviations from their target values.

From the government budget constraint above, \( l_t \) is defined as:

\[ l_t \equiv \frac{R_t B_t + Q_t S_{t+1} F_t^0}{P_t^H y_t^H} = \frac{R_t b_t + Q_t T T_{t+1}^{\nu + \nu'} - \frac{1}{\eta_{t+1}} \frac{\eta_t^{\tau_f}}{F_t^{\tau_f} P_t^{\tau_f}}}{T T_{t+1}^{\nu - 1} y_t^H} \] (356)

28 We focus on distorting policy instruments, because using lump-sum ones for shock and/or debt stabilization is like a free lunch.
where $\Pi_t \equiv \frac{\Delta P_t}{P_{t-1}}$ is the gross domestic CPI inflation rate, $\Pi_t^* \equiv \frac{P^*_t}{P_{t-1}}$ is the gross foreign CPI inflation rate, $TT_t \equiv \frac{P^*_t}{P_t} = \frac{S_t P^*_H}{P_t}$ is the terms of trade, $29$ and $f_t^T \equiv TT_t \frac{f_t^H}{f_t}^*$ is government debt issued in foreign currency and expressed in domestic currency in real terms (see Appendix for these transformations).

4.2.10 Exogenous variables and shocks

We assume that, along the transition, foreign imports, or equivalently domestic exports, are a function of terms of trade, where both variables are expressed as deviations from their long-run values:

$$\frac{c_t^F - c_t^{F*}}{c_t^{F*}} = \left( \frac{TT_t}{TT} \right)^\gamma$$

where $0 < \gamma < 1$ is a parameter. The idea is that foreign imports rise when the domestic economy becomes more competitive. Regarding the other rest-of-the-world variables, $Q_t^*$, $\Pi_t^{H*}$, we simply assume that they are constant over time and equal to $Q_t^* = 1.0303$ (which is as in the data - see below) and $\Pi_t^{H*} = 1$ at all $t$. We set the gross rate of exchange depreciation, $\epsilon_t$, at one, while, the output share of lump-sum taxes/transfers, $\{s_t^l\}_{t=0}^\infty$, and the output share of domestic public debt, $\{s_t^k\}_{t=0}^\infty$, are set respectively at $s_t^l = -0.21$ and $s_t^k = 0.66$ at all $t$, which are their data average values.

Finally, stochasticity comes from TFP, which is assumed to follow a stochastic AR(1) process:

$$\log (A_t) = (1 - \rho^a) \log (A) + \rho^a \log (A_{t-1}) + \epsilon_t^a$$

where $0 < \rho^a < 1$ is a parameter, variables without time subscript denote long-run values and $\epsilon_t^a \sim N(0, \sigma_{\epsilon_t^a}^2)$. We report that our main results do not change when we add extra shocks on the demand side of the economy.

4.2.11 Full equilibrium system (given feedback policy coefficients)

We now present the full equilibrium system. It consists of the 25 equations of the transformed DE presented in Appendix, the 4 policy rules and the definition for $l_t$ in subsection 4.2.9, and the equation for domestic exports in subsection 4.2.10. Thus, we have a system of 31 equations. By using 2 auxiliary variables, we transform it to a first-order, $30$ so we end up with 33 equations in 33 variables, $\{y_t^H, x_t, c_t^H, c_t^F, x_t, n_t, m_t, f_t^F, f_t^T, k_t, d_t, m_d, \Pi_t, \Pi_t^*, \Theta_t, \Delta_t, TT_t, w_t, r_t^k, Q_t, l_t, z_t^1, z_t^2, V_t, R_t, \tau_t^G, s_t^F, s_t^H, s_t^k, s_t^F, s_t^H, s_t^k, TTlag_t, c_t^{F*}\}_{t=0}^\infty$. This is given the exogenous variables, $\{Q_t^*, \Pi_t^{H*}, A_t, c_t, s_t^k, b_t^F, b_t^H\}_{t=0}^\infty$, which have been defined in subsection 4.2.10. The 33 endogenous variables are distinguished in 25 control variables, $\{y_t^H, c_t^H, c_t^F, x_t, n_t, d_t, \Pi_t, \Pi_t^*, \Theta_t, TT_t, w_t, r_t^k, l_t, z_t^1, z_t^2, V_t, \klead_t, c_t^{F*}, \tau_t^G, s_t^F, s_t^H, s_t^k, TTlag_t\}_{t=0}^\infty$, and 8 state variables, $\{m_{t-1}, f_{t-1}^F, f_{t-1}^T, k_{t-1}, \Delta_{t-1}, Q_{t-1}, R_{t-1}, TTlag_{t-1}\}_{t=0}^\infty$. All this is given the values of feedback policy coefficients in the policy rules defined in subsection 4.2.9.

To solve this system, we will take a second-order approximation around its long-run solution. We thus start with the long-run solution in the next section 4.3. In turn, we will study transition dynamics and the optimal choice of feedback policy coefficients along the transition.

$29$ Thus, $\frac{TT_t}{TT_{t-1}} = \frac{TT_t}{TT_{t-1}}^* = \frac{P^*_t}{P_{t-1}}$, where $\Pi_t^* \equiv \frac{P^*_t}{P_{t-1}}$.

$30$ In particular, we add 2 auxiliary endogenous variables, $\klead$ and $TTlag$, to reduce the dynamic system into a first-order one.
4.3 Data, parameterization and long-run solution

This section calibrates the model by using fiscal and public finance data from Italy over 2001-2011 and then presents the long-run solution. Recall that, since policy instruments react to deviations of macroeconomic indicators from their long-run target values, feedback policy coefficients do not play any role in the long-run solution.

4.3.1 Data and calibration

The fiscal and public finance data for Italy are from OECD Statistics and the Eurostat. The time unit is meant to be a year. The baseline parameter values, as well as the values of policy variables, are summarized in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.42</td>
<td>share of capital</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9603</td>
<td>time preference rate</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.5</td>
<td>home goods bias parameter</td>
</tr>
<tr>
<td>$\mu$</td>
<td>3.42</td>
<td>parameter related to money demand elasticity</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.04</td>
<td>capital depreciation rate</td>
</tr>
<tr>
<td>$\phi$</td>
<td>6</td>
<td>price elasticity of demand</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1</td>
<td>Frisch labour elasticity</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>elasticity of intertemporal substitution</td>
</tr>
<tr>
<td>$\nu^*$</td>
<td>0.5</td>
<td>foreign goods bias parameter</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.5</td>
<td>price rigidity parameter</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.05</td>
<td>risk premia parameter</td>
</tr>
<tr>
<td>$\chi_m$</td>
<td>0.001</td>
<td>preference parameter related to real money balances</td>
</tr>
<tr>
<td>$\chi_a$</td>
<td>7</td>
<td>preference parameter related to work effort</td>
</tr>
<tr>
<td>$\chi_g$</td>
<td>0.1</td>
<td>preference parameter related to public spending</td>
</tr>
<tr>
<td>$d$</td>
<td>0.9</td>
<td>threshold value for share of public debt</td>
</tr>
<tr>
<td>$\rho^a$</td>
<td>0.92</td>
<td>persistence of TFP</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.017</td>
<td>standard deviation of TFP</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.9</td>
<td>foreign imports parameter</td>
</tr>
<tr>
<td>$\xi$</td>
<td>2</td>
<td>adjustment cost parameter on physical capital</td>
</tr>
<tr>
<td>$\phi^g$</td>
<td>2</td>
<td>adjustment cost parameter on foreign public debt</td>
</tr>
<tr>
<td>$\phi^h$</td>
<td>2</td>
<td>adjustment cost parameter on international assets</td>
</tr>
<tr>
<td>$R$</td>
<td>1.0413</td>
<td>gross nominal interest rate</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>0.17</td>
<td>consumption tax rate</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>0.32</td>
<td>capital tax rate</td>
</tr>
<tr>
<td>$\tau^l$</td>
<td>0.42</td>
<td>labour tax rate</td>
</tr>
<tr>
<td>$s^g$</td>
<td>0.22</td>
<td>government spending as share of output</td>
</tr>
<tr>
<td>$s^p$</td>
<td>0.66</td>
<td>domestic public debt as share of output</td>
</tr>
<tr>
<td>$s^b + s^f$</td>
<td>1.1</td>
<td>domestic and foreign public debt as shares of output</td>
</tr>
</tbody>
</table>

The value of the time preference rate, $\beta$, follows from setting $R = 1.0413$ for the gross nominal interest rate (this implies a risk premium of 1.1% over the German 10-year bond rate, which is the average value in the data) and $\Pi = 1$ for the long-run gross inflation rate. The real money balances elasticity, $\mu$, is taken from Pappa and Neiss (2005). We use conventional values used by the literature for the elasticity of intertemporal substitution, $\sigma$, the inverse of Frisch...
labour elasticity, \( \eta \), and the price elasticity of demand, \( \phi \), which are all taken from Andrès and Doménech (2006) and Gali (2008). Regarding preference parameters in the utility function, \( \chi_m \) is chosen to obtain a yearly steady-state value for real money balances as ratio of output 0.48, \( \chi_n \) is chosen to obtain yearly steady-state labour hours 0.27, while \( \chi_g \) is set at 0.1. The price rigidity parameter, \( \theta \), is set at 0.5 (as we report below, we have experimented with various values of \( \theta \) and all key results remain unaffected).

In our baseline parameterization, the critical value of the output share of public debt above which sovereign risk premia emerge, \( \bar{d} \), is set at 0.9, which is consistent with evidence provided by Reinhart and Rogo¤ (2010) that, in advanced economies, there is no obvious link between public debt and macroeconomic performance until total public debt reaches the 90% threshold. The associated sovereign premium parameter, \( \psi \), is set at 0.05, which, jointly with the value of \( \bar{d} \), implies a steady-state premium for Italy over the German rate equal to 1.1%. These values are in line with empirical findings for OECD countries (see Ardagna et al., 2004). This parameterization means that, when public debt increases by 1 percentage point from its threshold value \( \bar{d} \), the nominal interest rate paid by the country increases by 5 basis points above its steady-state value. As we report below, our main results are robust to changes in these parameter values.

Concerning the exogenous stochastic variables, we set \( \rho^a = 0.92 \) and \( \sigma_a = 0.017 \) for persistence and standard deviation respectively of the TFP shock (the value of \( \rho^a \) is similar to that in Schnitt-Grohé and Uribe, 2007, while the value of \( \sigma_a \) is close to that in Bi, 2010, and Bi and Kumhof, 2009). As reported below, our results are robust to changes in these values. Regarding the rest-of-the world variables, \( \Pi^H_t, Q^*_t \) and \( c^F_t \), we set their long-run average values equal to \( \Pi^H_t = 1, Q^*_t = 1.0303 \) and \( c^F_t = 0.9c^F_t \), where 0.9 is calibrated to replicate the net export position found in the Italian data. The parameter \( \gamma \) in equation (357) for foreign imports is set at 0.9; in combination with the other parameters, this gives a dynamically stable solution.

The long-run values of fiscal and public finance policy instruments, \( \tau^c_t, \tau^b_t, \tau^v_t, s^d_t, s^f_t, s^b_t \), are either set at their data averages, or are calibrated to deliver data-consistent steady-state values for the residually determined fiscal variables. In particular, \( \tau^c_t, \tau^b_t, \tau^v_t \) are the effective tax rates on consumption, capital and labor in the data over 2001-2011. Moreover, \( s^b_t \equiv \frac{b^T_t}{y^T_t} = \frac{b^T_t}{y^T_t} \) and \( s^f_t \equiv \frac{s^f_t}{y^T_t}, \) namely, the domestic and foreign public debt-to-output ratios, are set at their average values in the Italian data, where the average of total public debt-to-output ratio, \( s^b_t + s^f_t \), is around 1.1 in the data.

4.3.2 Long-run solution or the "status quo"

Table 2 presents the long-run solution of the model economy when we use the parameter values and the policy instruments in Table 1. In this long-run solution, we treat \( s^f_t \) as the residually determined public finance instrument, while \( \tau^c_t, \tau^b_t, \tau^v_t, s^d_t, s^f_t, s^b_t \) are set at their data average values. In Table 2, we also present some key ratios in the Italian data whenever available. Most of the solved ratios are close to their actual values. Notice, in particular, our solution for total foreign debt as share of output, which is \( \frac{f^T_t - f^T_t}{y^T_t} \) is 0.36; its value in the data is very close, around 0.35 (see Diz Dias, 2010, for external debt statistics of the euro area).

This solution will serve as a point of departure. That is, in what follows, we depart from this solution to study various policy experiments. This is why we call it the "status quo" solution.
Table 2: Long-run solution (or the "status quo")

<table>
<thead>
<tr>
<th>Variables</th>
<th>Long-run values</th>
<th>Variables</th>
<th>Long-run values</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y^H$</td>
<td>0.59</td>
<td>$V$</td>
<td>16.2878</td>
<td>-</td>
</tr>
<tr>
<td>$c$</td>
<td>0.42</td>
<td>$\Pi^*$</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$c^H$</td>
<td>0.19</td>
<td>$z^s$</td>
<td>1.23</td>
<td>-</td>
</tr>
<tr>
<td>$c^F$</td>
<td>0.22</td>
<td>$z^z$</td>
<td>1.03</td>
<td>-</td>
</tr>
<tr>
<td>$n$</td>
<td>0.27</td>
<td>$l$</td>
<td>1.15</td>
<td>-</td>
</tr>
<tr>
<td>$x$</td>
<td>0.07</td>
<td>$\frac{1}{y^H}$</td>
<td>0.71</td>
<td>0.73</td>
</tr>
<tr>
<td>$f^h$</td>
<td>0.05</td>
<td>$\frac{1}{y^H}$</td>
<td>2.95</td>
<td>3.48</td>
</tr>
<tr>
<td>$m$</td>
<td>0.28</td>
<td>$TT^x$</td>
<td>0.08</td>
<td>0.1</td>
</tr>
<tr>
<td>$TT$</td>
<td>0.86</td>
<td>$\frac{1}{y^H}$</td>
<td>0.46</td>
<td>-</td>
</tr>
<tr>
<td>$w$</td>
<td>1.13</td>
<td>$s^t$</td>
<td>-0.21</td>
<td>-0.21</td>
</tr>
<tr>
<td>$mc$</td>
<td>0.9</td>
<td>$TT^x$</td>
<td>0.36</td>
<td>0.35</td>
</tr>
<tr>
<td>$d^H$</td>
<td>0.1</td>
<td>$\Pi$</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$r^x$</td>
<td>0.12</td>
<td>$\Pi^H$</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$Q$</td>
<td>1.0413</td>
<td>$\Theta$</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$k$</td>
<td>1.74</td>
<td>$\Delta$</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

In this solution, a lower public debt to output ratio implies a lower sovereign premium and this leads to higher capital, higher output and higher welfare. This can rationalize the debt consolidation policies studied below.

4.4 How we work

In this section, we explain the policy experiments we focus on, how we model debt consolidation and how we compare different optimized policy rules.

Recall that, along the transition path, nominal rigidities imply that money is not neutral so that monetary policy and the exchange rate regime matter to the real economy. As said in subsection 4.2.7, here we focus on fixed exchange rates and loss of monetary policy independence. Also, recall that, along the transition path, different counter-cyclical fiscal policy rules can have different implications. That is, we will welfare rank different counter-cyclical fiscal policy rules when there is no room for monetary policy.

4.4.1 Policy experiments

We will study two environments regarding policy action. In the first, used as a benchmark, the role of policy is only to stabilize the economy against temporary shocks. In particular, we assume that the economy is hit by an adverse temporary TFP shock, as defined in equation (358) above, which produces a contraction in output, a rise in the public debt to output ratio and a rise in the sovereign premium. Then, the policy questions are which policy instrument to use, and how strong the reaction of policy instruments to deviations from targets should be, where these targets are given by the status quo solution. Technically speaking, in this case, we depart from, and end up, at the same long run, which is the status quo in subsection 4.3.2 above, while transition dynamics are driven by temporary shocks only.

The second environment is richer. Now the role of policy is twofold: to stabilize the economy against the same shocks as above and, at the same time, to improve resource allocation by gradually reducing the public debt ratio over time. The policy questions are as above except that
now the policy targets are given by the long-run solution of the reformed economy. Technically speaking, in this case, we depart from the status quo solution, but we end up at a reformed long run with lower public debt and zero sovereign premia. Thus, now there are two sources of transition dynamics: temporary shocks and the difference between the initial and the new reformed long run (see also Cantore et al., 2012).

Although our interest is in the latter reformed economy, the former serves as a natural welfare benchmark. The next subsection provides the definition of debt consolidation adopted here.

4.4.2 How we model debt consolidation

In the reformed economy, the government reduces the share of public debt from 110% (which is its average value in the data over the sample period and is also our status quo solution) to the target value of 90%. Since, in our model, sovereign risk premia arise whenever public debt happens to be above the 90% threshold, premia are eliminated once such consolidation is achieved. Debt reductions are accommodated by adjustments in the tax-spending policy instruments, namely, the output share of public spending, and the tax rates on capital income, labour income and consumption.

It is recognized that debt consolidation implies a tradeoff between short-term pain and medium-term gain. During the early phase of the transition, debt consolidation comes at the cost of higher taxes and/or lower public spending. In the medium- and long-run, a reduction in the debt burden allows, other things equal, a cut in tax rates, and/or a rise in public spending. Thus, one has to value the early costs of stabilization vis-a-vis the medium- and long-term benefits from the fiscal space created.

It is also recognized that the implications of fiscal reforms, like debt consolidation, depend heavily on the public financing policy instrument used, namely, which policy instrument adjusts endogenously to accommodate the exogenous changes in fiscal policy (see e.g. Leeper et al., 2010, and Leeper, 2010). In the case of debt consolidation, such implications are expected to depend both on which policy instrument bears the cost of adjustment in the early period of adjustment and on which policy instrument is expected to reap the benefit, once consolidation has been achieved. Notice that if lump-sum policy instruments were available, the costs of adjustment, as well as the benefits after adjustment has been achieved, would be trivial.

To understand the logic of our results, and following usual practice in related studies, we will start by experimenting with one fiscal instrument at a time. This means that, along the early costly phase, we allow one of the fiscal policy instruments to react to public debt imbalances (so as to stabilize debt around its new target value of 0.9) and, at the same time, it is the same fiscal policy instrument that adjusts residually in the long-run to close the government budget. Thus, we will start by assuming that the same policy instrument bears the cost of, and reaps the benefit from, debt consolidation. In turn, we will experiment with fiscal policy mixes, which means that we can use different fiscal policy instruments in the transition and in the long run.

The rules for fiscal policy instruments are as in subsection 4.2.9 above except that now the targeted values are those of the reformed long-run equilibrium. In all experiments, all other fiscal policy instruments, except the one used for stabilization, remain unchanged and equal to their pre-reform status quo values.

In particular, we work as follows in the case with debt consolidation. We first solve and compare the long-run equilibria with, and without, debt consolidation. In turn, setting, as initial conditions for the state variables, their long-run values from the solution of the economy without debt consolidation (this is the status quo in subsection 4.3.2), we compute the equilibrium transition path as we travel to the long run of the reformed economy. This is for each method of
public financing used. The feedback policy coefficients of the instrument(s) used for stabilization along the transition path are chosen optimally. The way we compute optimized feedback policy rules, with or without debt consolidation, is explained in the next subsection.

4.4.3 How we compute optimized feedback policy rules

Irrespective of the policy experiments studied, to make the comparison of different policies meaningful, we compute optimized policy rules, so that results do not depend on ad hoc differences in feedback policy coefficients across different policy rules. The welfare criterion is household’s expected lifetime utility.

To do so, we work in two steps. In the first preliminary step, we search for the ranges of feedback policy coefficients, as defined in equations (352-355), which allow us to get a locally determinate equilibrium (this is what Schmitt-Grohé and Uribe, 2007, call implementable rules). If necessary, these ranges will be further restricted so as to give economically meaningful solutions for the policy instruments (e.g. tax rates less than one). In our search for local determinacy, we experiment with one, or more, policy instruments and one, or more, operating targets at a time.

In the second step, within the determinacy ranges found above, we compute the welfare-maximizing values of feedback policy coefficients (this is what Schmitt-Grohé and Uribe, 2005 and 2007, call optimized policy rules). The welfare criterion is to maximize the conditional welfare, $E_0 V_0$, as defined in (??) in the Appendix A, where conditionality refers to the initial conditions chosen; the latter are given by the status quo long-run solution. To this end, following e.g. Schmitt-Grohé and Uribe (2004), we take a second-order approximation to both the equilibrium conditions and the welfare criterion. As is well known, this is consistent with risk-averse behavior on the part of economic agents and can also help us to avoid possible spurious welfare results that may arise when one takes a second-order approximation to the welfare criterion combined with a first-order approximation to the equilibrium conditions (see e.g. Gali, 2008, pp. 110-111, Malley et al., 2009, and Benigno and Woodford, 2012).

In other words, we first compute a second-order accurate approximation of conditional welfare, and the associated decentralized equilibrium, as functions of feedback policy coefficients by using the perturbation method of Schmitt-Grohé and Uribe (2004) and, in turn, we use a matlab function (such as fminsearch.m or fminsearchbnd.m) to compute the values of the feedback policy coefficients that maximize the second-order accurate approximation of conditional welfare (matlab routines are available upon request). In this exercise, as said above, the feedback policy coefficients are restricted to be within some prespecified ranges delivering determinacy as well as meaningful values for policy instruments. All this is with, and without, debt consolidation.

4.5 Results from policy experiments

In this section, we present the main results. The emphasis will be on the case of the reformed economy but, for reasons of comparison, we also present results for the case without debt consolidation. We start by defining the region of feedback policy coefficients that can give local determinacy.

4.5.1 Determinacy areas

As is known, local determinacy depends crucially on the values of feedback policy coefficients. Our experiments show that economic policy can guarantee determinacy when fiscal policy instruments ($s^g_t$, $\tau^f_t$, $\tau^n_t$, $\tau^b_t$) react to public liabilities between critical minimum and maximum values, where these values differ across different policy instruments. In particular, the ranges are $0.05 < \gamma^g_t < 2.65$, $0.08 < \gamma^n_t < 11.02$, $0.1 < \gamma^b_t < 43.9$ and $0.12 < \gamma^f_t < 4.9$ for $s^g_t$, $\tau^f_t$, $\tau^n_t$ and
respectively. By contrast, the values of $\gamma_0^q$, where $q \equiv (g, c, k, n)$, measuring the reaction of fiscal policy instruments to the output gap, are not found to be critical to determinacy. Further details are available upon request.

### 4.5.2 Optimized policy rules and welfare results

Within the determinacy ranges, we now compute optimized policy rules. Results, for the case with debt consolidation, are reported in Table 3. The first column lists the fiscal policy instrument used, while the optimal reaction of this instrument to debt and output gaps are in the second column. The third column reports the standard deviation of the instrument used relative to that of output, as well as its minimum value during transition, if the instrument is the public spending share, or its maximum value during transition, if it is a tax rate (we include this information because we want to make sure that we get values that make sense economically and also do not differ substantially from those in the historical data; we also report that the resulting equilibrium nominal interest rate is above the zero bound in all solutions reported below). Long-run utility, $u$, is reported in the second column from the end, while expected discounted lifetime utility, $E_0V_0$, is reported in the last column.\(^{31}\) Recall that, under each policy regime, feedback policy coefficients are chosen to maximize $E_0V_0$, so welfare differences should not be expected to be large across policy regimes.

The results show that, if we rank instruments according to expected discounted lifetime utility, the labor tax rate comes first, the capital tax rate is second, then the consumption tax rate and, lastly, the share of public spending. Keep in mind that these results are in terms of lifetime utility because shorter time horizons may imply different things (see below). We thus prefer to postpone the interpretation of our welfare ranking until below.

It is worth noticing that, in all cases in Table 3, the optimal reaction to the output gap is smaller in magnitude than the optimal reaction to debt. Thus, the main policy concern is about debt imbalances rather than the recession (this is confirmed by impulse response functions below). Loosely speaking, fiscal activism needs to be treated with caution (see also e.g. Gordon and Leeper, 2003, Taylor, 2009, and Feldstein, 2009, who argue that counter-cyclical fiscal policy is counter-productive).

\(^{31}\)To compare regimes, we could also use a flat consumption subsidy that makes the agent indifferent between two regimes (see e.g. Lucas, 1990). The message will be the same.
Table 3: Optimized policy rules under debt consolidation and stabilization

<table>
<thead>
<tr>
<th>Fiscal instrument</th>
<th>Optimal reaction to debt and output</th>
<th>Volatility and max/min value of instrument</th>
<th>Long-run utility $u$</th>
<th>Expected discounted lifetime utility $E_0 V_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^g_t$</td>
<td>$\gamma^g_t = 0.18$ $\gamma^g_y = 0.07$</td>
<td>$\frac{\text{std}(s^g_t)}{\text{std}(y_t)} = 0.45$ $\frac{s^g_t}{y_t} = 0.17$ $R_t = 1.0723$</td>
<td>0.7323</td>
<td>22.9178</td>
</tr>
<tr>
<td>$\tau^c_t$</td>
<td>$\gamma^c_t = 0.51$ $\gamma^c_y = 0.23$</td>
<td>$\frac{\text{std}(\tau^c_t)}{\text{std}(y_t)} = 1.5291$ $\frac{\tau^c_t}{y_t} = 0.66$ $R_t = 1.0745$</td>
<td>0.7329</td>
<td>23.0222</td>
</tr>
<tr>
<td>$\tau^k_t$</td>
<td>$\gamma^k_t = 0.45$ $\gamma^k_y = 0.15$</td>
<td>$\frac{\text{std}(\tau^k_t)}{\text{std}(y_t)} = 0.795$ $\frac{\tau^k_t}{y_t} = 0.57$ $R_t = 1.0723$</td>
<td>0.7721</td>
<td>23.8841</td>
</tr>
<tr>
<td>$\tau^n_t$</td>
<td>$\gamma^n_t = 0.43$ $\gamma^n_y = 0.15$</td>
<td>$\frac{\text{std}(\tau^n_t)}{\text{std}(y_t)} = 0.6436$ $\frac{\tau^n_t}{y_t} = 0.5842$ $R_t = 1.0723$</td>
<td>0.7597</td>
<td>23.9025</td>
</tr>
</tbody>
</table>

4.5.3 Impulse response functions under optimized policy rules

We now present the associated impulse response functions (IRFs) of some key endogenous variables (recall that transition dynamics are driven by a temporary adverse supply shock and by debt consolidation as explained above). Results for the four fiscal policy instruments, $s^g_t$, $\tau^c_t$, $\tau^k_t$ and $\tau^n_t$, are shown in Figures 1a-1d respectively. Since, in all cases, it is optimal for policy instruments to react to more than one indicator at the same time, the IRFs are useful to illustrate which reaction dominates and what drives the change in the policy instrument over time. Variables are expressed as log-deviations from their new, reformed long-run values (while, we depart from the status quo).

Figure 1a starts with the case in which we use the share of public spending, $s^g_t$, as the state-contingent instrument. That is, following Table 3, row 1, $s^g_t$ reacts to public debt and output with feedback coefficients $\gamma^g_t = 0.18$ and $\gamma^g_y = 0.07$ respectively, while all other policy feedback coefficients are set at zero meaning that the other policy instruments, $\tau^c_t$, $\tau^k_t$ and $\tau^n_t$, remain constant at their steady-state values (data averages). Figures 1b, 1c and 1d do the same when we use $\tau^c_t$, $\tau^k_t$ and $\tau^n_t$ respectively as state-contingent policy instruments.
Figure 1a: IRFs when the fiscal instrument is government spending

Figure 1b: IRFs when the fiscal instrument is the consumption tax rate
Figures 1a-d imply that public spending should fall, and tax rates should rise. In other words, the concern for debt consolidation more than offsets the concern for shock stabilization, even when the economy is hit an adverse supply shock that triggers a recession. Thus, changes in all fiscal instruments are driven by debt imbalances most of the time.
To make our results clearer, Figure 2 also presents the implied total public debt as share of output (now, this is shown as a rate, rather than as deviation of this rate from its long-run value, which was the case in the IRFs above). As can be seen, debt starts at 110%, which is its status quo value, and ends up at 90% at the new reformed long run. In the very short run, the economy is hit by an adverse shock that reduces output and increases the debt share but then, thanks to active fiscal consolidation, debt starts falling towards its 90% threshold.

Inspection of Figures 1 and 2 implies that the duration of the debt consolidation phase, or equivalently the speed of debt reduction, depend on which fiscal instrument used. If we use the consumption tax rate, $\tau^c_t$, it should take around six time periods or years only; if we use the public spending ratio, $s^g_t$, or the capital tax rate, $\tau^k_t$, around ten years; finally, adjustment should be slower, around twelve years, if we use the labor tax rate, $\tau^n_t$. The idea is that the more distorting the instrument is, the slower the adjustment should be.

Figure 2: Path of public debt as share of output (in levels)

Note: The black solid line is the reformed long-run value in which total public debt converges to 90%.

4.5.4 Welfare over various time horizons

We now study what happens to welfare over various time horizons. This is important because, for several (political-economy) reasons, economic agents’ behavior can be short termist or impatient. Setting the feedback policy coefficients as in Table 3 above, the expected discounted utility at various time horizons is reported in Table 4. Numbers in parentheses report results without debt consolidation, other things equal (as explained above, without debt consolidation, we again
compute optimized feedback policy rules but now the economy starts from, and also returns to, its status quo).

There are, at least, two messages from Table 4. First, other things equal, debt consolidation improves welfare only if we are relatively far-sighted. Our results imply that expected discounted utility is higher with, than without, debt consolidation, if we care beyond the first 20, 20, 16 and 14 time periods (this is when we use $s^g$, $\tau^c$, $\tau^k$ and $\tau^n$ respectively). Reversing the argument, debt consolidation comes at a short term cost: expected discounted utility is higher without debt consolidation, if we care only about the first 20, 20, 16 and 14 time periods (this is when we use $s^g$, $\tau^c$, $\tau^k$ and $\tau^n$ respectively). Thus, as it happens with most reforms, the argument for, or against, debt consolidation involves a value judgment.

Second, without debt consolidation, and to the extent that feedback policy coefficients are optimally chosen, the choice of the fiscal policy instrument used for cyclical stabilization is not so important. Welfare differences appear after the third decimal point across all time horizons (these are the numbers in parentheses). By contrast, with debt consolidation, the choice of the fiscal policy instrument matters (these are the numbers without parentheses). Labor and capital taxes score better than public spending and consumption taxes over all time horizons.

The relative superiority of labor and capital taxes may look surprising, at least at first sight. In particular, it may look surprising that the best fiscal instrument is not public spending. The latter is what one would expect from policy reports (see e.g. European Commission, 2011) and related closed economy studies (see e.g. Philippopoulos et al., 2013).

To understand our results, recall that here we have a small open economy with sovereign premia. These premia appear in the Euler equation for foreign assets and introduce extra distortions/wedges in relative prices. Our results then imply that labor and capital taxes are more suitable to address these extra distortions/wedges during the early costly period of debt reduction (see also the general discussion in Wren-Lewis, 2010). In turn, once sovereign debt and premia have been reduced and fiscal space has been created, the most efficient way to take advantage of this fiscal space is to reduce labor and capital taxes, which are particularly distorting. The combination of these two effects (one in the early phase of fiscal pain and the other in the later phase of fiscal gain) makes the labor and capital taxes more desirable than public spending and consumption taxes.

In a closed economy, by contrast, although it is again better to allow the labor and capital taxes to take advantage of the fiscal space in the later phase of fiscal gain, as it happens in the small open economy, it is better to use a less distorting policy instrument, like public spending, in the early phase, and it is what happens in the early phase that shapes lifetime utility quantitatively (see Philippopoulos et al., 2013).

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It should be pointed out that the rise in welfare is partly driven by the fact that debt consolidation and elimination of sovereign premia in the reformed long-run equilibrium allow a higher value of the time preference rate than in the pre-reformed long-run solution in section 3 (in particular, the calibrated value of $\beta$ was 0.9603 in the status quo solution in section 3, while it is 0.9709 without premia).
Table 4: Welfare at different time horizons with, and without, debt consolidation

<table>
<thead>
<tr>
<th></th>
<th>4 periods</th>
<th>10 periods</th>
<th>50 periods</th>
<th>lifetime</th>
<th>long run</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^g_t$</td>
<td>1.9226</td>
<td>17.2934</td>
<td>22.9178</td>
<td>0.7323</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.5098)</td>
<td>(14.5509)</td>
<td>(16.2654)</td>
<td>(0.6466)</td>
<td></td>
</tr>
<tr>
<td>$\tau^c_t$</td>
<td>1.7027</td>
<td>17.0326</td>
<td>23.0222</td>
<td>0.7329</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.5098)</td>
<td>(14.5520)</td>
<td>(16.2670)</td>
<td>(0.6466)</td>
<td></td>
</tr>
<tr>
<td>$\tau^k_t$</td>
<td>1.9259</td>
<td>17.9245</td>
<td>23.8841</td>
<td>0.7721</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.5096)</td>
<td>(14.5516)</td>
<td>(16.2671)</td>
<td>(0.6466)</td>
<td></td>
</tr>
<tr>
<td>$\tau^n_t$</td>
<td>2.0484</td>
<td>17.8414</td>
<td>23.9025</td>
<td>0.7597</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.5096)</td>
<td>(14.5537)</td>
<td>(16.2696)</td>
<td>(0.6466)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Results without debt consolidation in parentheses.

4.5.5 Implications for output

So far, we have focused on welfare implications. This was natural since our results are based on those values of feedback policy coefficients that maximize welfare. But what are the implications of such a policy for output? Results for output are reported in Table 5. Table 5 is like Table 4, except that now the endogenous variable reported is expected discounted output instead of expected discounted utility. Also, Figures 3 and 4 show, respectively, the time path of output and consumption under alternative policy rules (as levels, not as log deviations from the new reformed long run). Labor and capital taxes seem again to be a better way of bringing debt down. Therefore, welfare-maximizing policy does not differ from output-maximizing policy in our model.

Table 5: Discounted output at different time horizons with, and without, debt consolidation

<table>
<thead>
<tr>
<th></th>
<th>4 periods</th>
<th>10 periods</th>
<th>50 periods</th>
<th>lifetime</th>
<th>long run</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^g_t$</td>
<td>2.2342</td>
<td>16.7759</td>
<td>22.0246</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.2222)</td>
<td>(12.9104)</td>
<td>(14.8754)</td>
<td>(0.59)</td>
<td></td>
</tr>
<tr>
<td>$\tau^c_t$</td>
<td>2.2149</td>
<td>16.3908</td>
<td>21.4847</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.2225)</td>
<td>(12.9130)</td>
<td>(14.8788)</td>
<td>(0.59)</td>
<td></td>
</tr>
<tr>
<td>$\tau^k_t$</td>
<td>2.2482</td>
<td>16.8769</td>
<td>22.2600</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.2223)</td>
<td>(12.9091)</td>
<td>(14.8739)</td>
<td>(0.59)</td>
<td></td>
</tr>
<tr>
<td>$\tau^n_t$</td>
<td>2.2455</td>
<td>16.9617</td>
<td>22.3524</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.2227)</td>
<td>(12.9102)</td>
<td>(14.8751)</td>
<td>(0.59)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Results without debt consolidation in parentheses.

Notice that in Figures 3 and 4, the black solid line shows the status-quo long-run value, i.e. 0.59 for output and 0.42 for consumption, respectively. Output and consumption converge to their new reformed long-run values, which are different for each fiscal instrument used. In particular, the reformed long-run values of output are 0.67, 0.65, 0.69 and 0.69 for $s^g$, $\tau^c$, $\tau^k$ and $\tau^n$ respectively, while, in Figure 4, the reformed long-run values of consumption are 0.45, 0.44, 0.46 and 0.46 for $s^g$, $\tau^c$, $\tau^k$ and $\tau^n$ respectively.
4.5.6 Policy mixes

So far, we have studied one fiscal instrument at a time. In particular, as explained in subsection 4.4.2 above, we have restricted ourselves to the case in which the same fiscal instrument is used
during the early phase of fiscal pain as well as during the later phase of fiscal gain. We now study policy mixes in the sense that we are now free to use different policy instruments in the transition and in the long run. Some interesting mixes, all with debt consolidation, are reported in Table 6.

A comparison of the results in Tables 4 and 6, implies that policy mixes lead to higher welfare, at least when we look at long-time horizons. In terms of the best policy mix, Table 6 implies that, if we care about the short- and medium-run, the best mix is to raise the labor tax rate during the early consolidation phase and to reduce the capital tax rate once fiscal space is created (this is the \(\tau^n - \tau^k\) combination in the third row of Table 6). Thus, the main results are as above when we had to use the same instrument all the time. Only if we care about long-time horizons, it is better to use the consumption tax rate during the early consolidation phase and the capital tax rate in the long run (this is the \(\tau^c - \tau^k\) combination in the first row of Table 6).

The idea now is that when results are dominated by what happens in the longer run, it is better to bring debt down as soon as possible, and this can be achieved by high consumption taxes in the short run, and then enjoy the benefits of expected cuts in distorting taxes, namely, ex ante cuts in capital taxes. All these results combined confirm how important expectations are in case we are far-sighted: expectations of cuts in distorting taxes, made affordable by the fiscal space created by a smaller debt burden, play a key role in the choice of the policy mix when we are far-sighted enough.

Table 6: Welfare at different time horizons with debt consolidation when we use fiscal policy mixes

<table>
<thead>
<tr>
<th>Policy instrument in the transition</th>
<th>Policy instrument in the long run</th>
<th>4 periods</th>
<th>10 periods</th>
<th>50 periods</th>
<th>lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau^c)</td>
<td>(\tau^k)</td>
<td>1.8587</td>
<td>5.1250</td>
<td>18.1554</td>
<td>24.1486</td>
</tr>
<tr>
<td>(s^n)</td>
<td>(\tau^k)</td>
<td>1.9657</td>
<td>5.1041</td>
<td>18.0882</td>
<td>24.0774</td>
</tr>
<tr>
<td>(\tau^n)</td>
<td>(\tau^k)</td>
<td>2.0174</td>
<td>5.1951</td>
<td>18.0756</td>
<td>24.0392</td>
</tr>
<tr>
<td>(\tau^c)</td>
<td>(\tau^n)</td>
<td>1.8989</td>
<td>5.1329</td>
<td>17.9037</td>
<td>23.9896</td>
</tr>
<tr>
<td>(s^n)</td>
<td>(\tau^n)</td>
<td>2.0016</td>
<td>5.1326</td>
<td>17.8559</td>
<td>23.9373</td>
</tr>
</tbody>
</table>

4.5.7 Robustness checks

Our results are robust to several checks. For instance, they are robust to adding more indicators in the feedback policy rules (like inflation or terms of trade) and to assuming a more volatile economy (by increasing the standard deviation of the TFP shock or by adding demand shocks). More importantly, they are robust to changes in the key parameter values. Among the latter, we have extensively experimented with changes in the values of the parameter in the sovereign premium equation, \(\psi\), the parameter in the exports function, \(\gamma\), and the Calvo parameter in the firm’s problem, \(\theta\), whose values are relatively unknown. We report that that our main results do not change within 0.002 \(\leq \psi \leq 0.09\), 0.5 \(\leq \gamma \leq 1\) and 0.1 \(\leq \theta \leq 0.5\). All these robustness checks are available upon request.

4.6 Concluding remarks and possible extensions

This paper has studied fiscal policy in a New Keynesian model of a semi-small open economy facing endogenous sovereign premia and not being able to use monetary policy. The focus has been on optimized simple and implementable policy rules for various categories of taxes and public spending.
Since the results are written in the Introduction, we close with some possible extensions. It would be interesting to add heterogeneity both in terms of economic agents within a country and in terms of countries. In particular, we could distinguish between private and public employees and so study the distributional implications of the stabilization policies studied here. It is also interesting to use a two-country model, where countries can differ in, say, fiscal imbalances and/or preferences and so study the cross-border effects of national stabilization and debt consolidation policies. We leave these extensions for future work.

4.7 Appendix

4.7.1 Households

This Appendix presents the problem of the household in some detail. There are $i = 1, 2, \ldots, N$ identical domestic households.

Household’s problem Each household $i$ maximizes expected lifetime utility:

$$ E_0 \sum_{t=0}^{\infty} \beta^t U (c_{i,t}, n_{i,t}, m_{i,t}, g_t) $$

where $c_{i,t}$ is $i$’s consumption bundle as defined above, $n_{i,t}$ is $i$’s hours of work, $m_{i,t} \equiv \frac{M_{i,t}}{P_t}$ is $i$’s real money holdings, $g_t$ is per capita public spending, $0 < \beta < 1$ is the time discount rate, and $E_0$ is the rational expectations operator conditional on the information set.

The period utility function is of the form (see also e.g. Gali, 2008):

$$ u_{i,t} (c_{i,t}, n_{i,t}, m_{i,t}, g_t) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_{i,t}^{1+\eta}}{1+\eta} + \chi_m \frac{m_{i,t}^{1-\mu}}{1-\mu} + \chi_g \frac{g_t^{1-\zeta}}{1-\zeta} $$

where $\chi_n$, $\chi_m$, $\chi_g$, $\sigma$, $\eta$, $\mu$, $\zeta$ are preference parameters.

The period budget constraint of each household $i$ expressed in real terms is:

$$(1 + \tau_{i,t}) \left[ \frac{P_t H_i}{P^* t} c_{i,t} + \frac{P_t F_i}{P^* t} f_{i,t} + \frac{P_t L_i}{P^* t} L_{i,t} + \frac{P_t L_i}{P^* t} L_{i,t} + \frac{S_t P_t^*}{P_t} f_{i,t} + \phi^h \frac{S_t P_t^*}{P_t} f_{i,t} - SP^* f^h \right]$$

$$= \left[ (1 - \tau_{i,t}^h) \left( \frac{P_t H_i}{P^* t} k_{i,t-1} + d_{i,t} \right) + (1 - \tau_{i,t}^-) w_t n_{i,t} + R_{t-1} \frac{P_t}{P^* t} b_{i,t-1} \right]$$

$$+ \frac{P_{t-1}}{P_t} m_{i,t-1} + Q_{t-1} \frac{S_t P_t^*}{P_t} P_t \frac{f_{i,t-1}^h - \tau_{i,t}^h}{P_t} $$

where $x_{i,t}$ is $i$’s domestic investment, $b_{i,t}$ is $i$’s end-of-period real domestic bonds, $m_{i,t}$ is $i$’s end-of-period real domestic money holdings, $f_{i,t}^h$ is $i$’s end-of-period real internationally traded assets denominated in foreign currency, $r_t^h$ is the real return to inherited domestic capital, $k_{i,t-1}$, $d_{i,t}$ is $i$’s real dividends received by domestic firms, $w_t$ is the real wage rate, $R_{t-1} \geq 1$ is the gross nominal return to domestic government bonds between $t-1$ and $t$, $Q_{t-1} \geq 1$ is the gross nominal return to international assets between $t-1$ and $t$, $\tau_{i,t}^h$, $\tau_{i,t}^\tau$, $\tau_{i,t}^\mu$ are tax rates on consumption, capital income and labour income respectively. Thus, small letters denote real variables, namely, $m_{i,t} \equiv \frac{M_{i,t}}{P_t}$, $b_{i,t} \equiv \frac{B_{i,t}}{P_t}$, $f_{i,t}^h \equiv \frac{F_{i,t}^h}{P_t}$, $w_t \equiv \frac{W_t}{P_t}$, $d_{i,t} \equiv \frac{D_{i,t}}{P_t}$, $\tau_{i,t}^h \equiv \frac{T_{i,t}^h}{P_t}$, where capital letters denote nominal variables. The parameter $\phi^h \geq 0$ captures transaction costs related to foreign assets, where
variables without time subscripts denote long-run values (these costs are not important to the main results but help the model with the data - see also below).

The law of motion of physical capital for household $i$ is:

$$k_{i,t} = (1 - \delta)k_{i,t-1} + x_{i,t} - \frac{\xi}{2} \left( \frac{k_{i,t}}{k_{i,t-1}} - 1 \right)^2 k_{i,t-1}$$

(362)

where $0 < \delta < 1$ is the depreciation rate of capital and $\xi \geq 0$ is a parameter capturing adjustment costs related to physical capital.

**Household’s optimality conditions** Each household $i$ acts competitively taking prices and policy as given. Following the literature, to solve the household’s problem, we follow a two-step procedure. We first suppose that the household determines its desired consumption of composite goods, $c^H_i(t)$ and $c^F_i(t)$, and, in turn, chooses how to distribute its purchases of individual varieties, $c^H_i(h)$ and $c^F_i(f)$.

The first-order conditions of each $i$ include the budget constraints and also:

$$\frac{\partial u_{i,t}}{\partial c_{i,t}} = \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \frac{P_t}{P_{t+1}}$$

(363)

$$\beta E_t \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \frac{P_t}{P_{t+1}} P_t S^F_{i,t} \left[ 1 + \phi \left( \frac{S^H_{i,t}}{P_t} f_i^h - \frac{S^F_{i,t}}{P_t} f_i^f \right) \right]$$

(364)

$$= \beta E_t \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \frac{P_t}{P_{t+1}} \frac{1}{P_{t+1}} \left( 1 - \xi \left( \frac{k_{i,t}}{k_{i,t-1}} - 1 \right) \right)$$

$$= \beta E_t \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \frac{P_t}{P_{t+1}} \frac{1}{P_{t+1}} \left( 1 - \xi \left( \frac{k_{i,t}}{k_{i,t-1}} - 1 \right) \right)$$

$$+ \xi \left( \frac{k_{i,t}}{k_{i,t-1}} - 1 \right)$$

(365)

$$\frac{\partial u_{i,t}}{\partial m_{i,t}} = \frac{\partial u_{i,t}}{\partial c_{i,t}} \frac{P_t}{P_{t+1}} P_t S^H_{i,t} \left[ 1 + \phi \left( \frac{S^H_{i,t}}{P_t} f_i^h - \frac{S^F_{i,t}}{P_t} f_i^f \right) \right]$$

(366)

$$\beta E_t \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \frac{P_t}{P_{t+1}} P_t S^H_{i,t} \left[ 1 + \phi \left( \frac{S^H_{i,t}}{P_t} f_i^h - \frac{S^F_{i,t}}{P_t} f_i^f \right) \right]$$

(367)

$$\frac{\partial c^H_{i,t}}{\partial \frac{P_t}{P^H}} = \frac{\nu}{1 - \nu} P_t$$

(368)

$$\frac{\partial c^F_{i,t}}{\partial \frac{P_t}{P^H}} = \frac{\nu}{1 - \nu} P_t$$

(369)

$$c^H_{i,t} = \left[ \frac{P_t}{P^H} \right] ^{-\phi} c^H_{i,t}$$

(370)

Equations (363)-(365) are respectively the Euler equations for domestic bonds, foreign assets and domestic capital, (366) is the optimality condition for money balances and (367) is the optimality condition for work hours. Finally, (368) shows the optimal allocation between domestic and foreign goods, while (369) and (370) show the optimal demand for each variety of domestic and foreign goods respectively.
Implications for price bundles  Equations (368), (369) and (370), combined with (336), (337) and (338), imply that the three price indexes are (see also e.g. Wickens, 2008, chapter 7):

\[ P_t = (P_t^H)\nu (P_t^F)^{1-\nu} \]  

(371)

\[ P_t^H = \left[ \sum_{h=1}^{N} \lambda_h [P_t^H(h)]^{1-\phi} \right]^{1/1-\phi} \]  

(372)

\[ P_t^F = \left[ \sum_{f=1}^{N} \lambda_f [P_t^F(f)]^{1-\phi} \right]^{1/1-\phi} \]  

(373)

4.7.2 Firms

This Appendix presents the problem of the firm in some detail. There are \( h = 1, 2, \ldots, N \) domestic firms. Each firm \( h \) produces a differentiated good of variety \( h \) under monopolistic competition facing Calvo-type nominal fixities.

Demand for firm’s product  Each domestic firm \( h \) faces demand for its product, \( y_t^H(h) \), coming from domestic households’ consumption and investment, \( c_t^H(h) \) and \( x_t(h) \), where \( c_t^H(h) \equiv \sum_{i=1}^{N} c_{i,t}^H(h) \) and \( x_t(h) \equiv \sum_{i=1}^{N} x_{i,t}(h) \), from the domestic government, \( g_t(h) \), and from foreign households’ consumption, \( c_t^{F*}(h) \equiv \sum_{i=1}^{N} c_{i,t}^{F*}(h) \). Thus, the demand for each domestic firm’s product is:

\[ y_t^H(h) = c_t^H(h) + x_t(h) + g_t(h) + c_t^{F*}(h) \]  

(374)

where, as in (369-370), we have:

\[ c_t^H(h) = \left( \frac{P_t^H(h)}{P_t^H} \right)^{-\phi} c_t^H \]  

(375)

\[ x_t(h) = \left( \frac{P_t^H(h)}{P_t^H} \right)^{-\phi} x_t \]  

(376)

\[ g_t(h) = \left( \frac{P_t^H(h)}{P_t^H} \right)^{-\phi} g_t \]  

(377)

\[ c_t^{F*}(h) = \left( \frac{P_t^{F*}(h)}{P_t^{F*}} \right)^{-\phi} c_t^{F*} \]  

(378)

where, using the law of one price discussed above, we have in (378):

\[ \frac{P_t^{F*}(h)}{P_t^{F*}} = \frac{P_t^H(h)}{P_t^H} = \frac{P_t^H(h)}{P_t^H} \]  

(379)

Since, at the economy level, aggregate demand for domestically produced goods is:

\[ y_t^H = c_t^H + x_t + g_t + c_t^{F*} \]  

(380)

the above equations imply that the demand for each domestic firm’s product is:

\[ y_t^H(h) = c_t^H(h) + x_t(h) + g_t(h) + c_t^{F*}(h) = \left( \frac{P_t^H(h)}{P_t^H} \right)^{-\phi} y_t^H \]  

(381)
**Firm’s problem**  Each domestic firm $h$ maximizes nominal profits, $D_t(h)$, defined as:

$$D_t(h) = P_t^H(h)y_t^H(h) - r_t^k P_t^H(h)k_{t-1}(h) - W_t n_t(h)$$  \hspace{1cm} (382)

All firms use the same technology represented by the production function:

$$y_t^H(h) = A_t[k_{t-1}(h)]^a [n_t(h)]^{1-a}$$  \hspace{1cm} (383)

where $A_t$ is an exogenous stochastic TFP process whose motion is defined below.

Since the firm operates under imperfect competition, profit maximization is subject to the demand for its product:

$$y_t^H(h) = P_t^H(h)y_t^H(h)$$  \hspace{1cm} (384)

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In each period, firm $h$ faces an exogenous probability $\theta$ of not being able to reset its price. A firm $h$, which is able to reset its price, chooses its price $P_{t}^{#}(h)$ to maximize the sum of discounted expected nominal profits for the next $k$ periods in which it may have to keep its price fixed.

**Firm’s optimality conditions**  To solve the firm’s problem, we follow a two-step procedure. We first solve a cost minimization problem, where each firm $h$ minimizes its cost by choosing factor inputs given technology and prices. The solution will give a minimum nominal cost function, which is a function of factor prices and output produced by the firm. In turn, given this cost function, each firm, which is able to reset its price, solves a maximization problem by choosing its price.

The solution to the cost minimization problem gives the input demand functions:

$$w_t = mc_t (1 - a) \frac{y_t(h)}{n_t(h)}$$  \hspace{1cm} (385)

$$\frac{P_t^H}{r_t^k} = mc_t a \frac{y_t(h)}{k_{t-1}(h)}$$  \hspace{1cm} (386)

where $mc_t$ is real marginal cost.

Then, the firm chooses its price to maximize nominal profits written as:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} D_{t+k}(h) = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P_{t}^{#}(h) y_{t+k}^H(h) - \Psi_{t+k}(y_{t+k}^H(h)) \right\}$$

where $\Xi_{t,t+k}$ is a discount factor taken as given by the firm, $y_{t+k}^H(h) = \left[ \frac{P_{t+k}^H}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H$ and $\Psi_{t}(\cdot)$ denotes the minimum nominal cost function for producing $y_{t}^H(h)$ at $t$ so that $\Psi'_{t}(\cdot)$ is the associated marginal cost.

The first-order condition gives:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P_{t}^{#}(h)}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H(h) \left\{ P_{t+k}^{#}(h) - \frac{\phi}{\phi - 1} \Psi'_{t+k} \right\} = 0$$  \hspace{1cm} (387)

We transform the above equation by dividing by the domestic aggregate price index, $P_t^H$.
\[
E_t \sum_{k=0}^{\infty} \theta^k [\Xi_{t+k} \left( \frac{P^H_t (h)}{P^H_{t+k}} \right)^{-\phi} \gamma_{t+k} \left( \frac{P^H_t (h)}{P^H_{t+k}} - \frac{\phi}{\phi - 1} \frac{m_{c_{t+k}} P_{t+k}}{P^H_{t+k}} \right)] = 0
\]

Therefore, the behaviour of each firm \( h \) is summarized by (385), (386) and (388). A recursive expression of this problem is presented below.

Notice that each firm \( h \), which can reset its price in period \( t \), solves an identical problem, so \( P^H_t (h) = P^h_t \) is independent of \( h \), and each firm \( h \), which cannot reset its price, just sets its previous period price \( P^H_t (h) = P^H_{t-1} (h) \). Thus, the evolution of the aggregate price level is given by:

\[
(P^H_t)^{1-\phi} = \theta (P^H_{t-1})^{1-\phi} + (1 - \theta) (P^H_t)^{1-\phi}
\]

### 4.7.3 Modelling fixed exchange rates

The related literature on exchange rate regimes (see e.g. Monacelli, 2004, Gali and Monacelli, 2005, Benigno et al., 2007, Benigno and Benigno, 2008) starts by assuming an independent monetary authority which sets the nominal interest rate, \( R_t \), according to a Taylor-type rule according to which the interest rate also feeds back on the gross depreciation rate, \( \epsilon_t \). Then, they model the fixed exchange rate regime as a polar case of that rule by setting a very large value for the feedback coefficient on \( \epsilon_t \); practically, a value that makes \( \epsilon_t \) close to one. This is like having fixed exchange rates in equilibrium or \textit{ex post} (see also the discussion in Himmels and Kirsanova, 2009). In our paper, to have fixed exchange rates both \textit{ex post} and \textit{ex ante}, we follow a more direct modelling approach. Under a fixed exchange rate regime, we set \( \epsilon_t \) exogenously at unity. Then, as mentioned above, we have to make one of the other policy instruments, \( R_t \), \( \tau^k \), \( \tau^{k^2} \), \( \tau^b \), \( \tau^l \), \( s^b \), \( s^l \), \( b_t \), endogenous to close the model. We have experimented with two choices: In the first, the endogenous variable is \( R_t \). In the second, one of \( s^b \), \( s^l \) or \( b_t \) is the endogenous variable. The former case is closer to being a member of a currency union, in the sense that the country keeps the exchange rate fixed and, at the same time, sacrifices monetary policy independence. The latter case is the textbook case of a country that fixes its currency but, thanks to imperfect capital mobility (here, generated by the endogenous world interest rate), can still use its monetary policy. Our numerical solutions below imply that in the latter case, namely if we try to use \( b_t \), \( s^l \) or \( s^b \) as the endogenous variable, we always come up with an unstable equilibrium. Thus, in what follows, we report results only for the case in which \( R_t \) is endogenous, namely we lose monetary policy independence, with the nominal domestic interest rate being determined by the world interest rate (which is exogenous) and the country’s risk premium (which is endogenous).

### 4.7.4 Decentralized equilibrium (given policy)

The Decentralized Equilibrium (DE) is summarized by the following 22 equations (quantities are in per capita terms):

\[
\frac{\partial u_t}{\partial c_t} \frac{1}{\frac{1}{1 + \tau_t^P} \frac{P^H_t}{P_{t+1}^P}} = \beta E_t \frac{\partial u_{t+1}}{\partial c_{t+1}} \frac{1}{\frac{1}{1 + \tau_{t+1}^P} \frac{P^H_{t+1}}{P_{t+1}^P}} R_t \frac{P_t}{P_{t+1}}
\]

\[
\beta \frac{\partial u_t}{\partial c_t} \frac{1}{\frac{1}{1 + \tau_t^P} \frac{P^H_t}{P_{t+1}^P}} \frac{S_t}{P_t} \frac{P^*_t}{P_t} \left( 1 + \phi \right) \frac{S_t}{P_t} \left( \frac{P^*_t}{P_t} - \frac{S_t}{P_t} \right)
\]

\[
\beta \frac{\partial u_t}{\partial c_t} \frac{1}{\frac{1}{1 + \tau_t^P} \frac{P^H_t}{P_{t+1}^P}} \frac{S_t}{P_t} \frac{P^*_t}{P_t} \left( Q_t \frac{S_t}{P_t} \frac{P^*_t}{P_t} - \frac{S_t}{P_t} \right)
\]

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\[
E_t \sum_{k=0}^{\infty} \theta^k [\Xi_{t,t+k}] \left[ \frac{P_t^{\#}}{P_{t+k}} \right]^{-\phi} y_{t+k} \left\{ \frac{P_t^{\#}}{P_t} - \frac{\phi}{\phi - 1} mc_{t+k} P_{t+k} \right\} = 0
\]

\[
y_{t+1} = \frac{1}{\phi - 1} \sum_{i=0}^{\infty} A_i k_{t-i} n_t^{1-a}
\]

\[
\begin{align*}
&b_t + m_t + S_{t-1} f f_t^s + \frac{p_t}{P_{t-1}} f f_t^s + \frac{p_t}{P_{t-1}} c f_t^s + \frac{\phi}{\phi - 1} \left( \frac{S_t f f_t^s}{P_t} - \frac{S_t c f_t^s}{P_t} \right)^2 + \frac{R_{t-1} b_{t-1}}{n_t - 1} + \frac{m_{t-1}}{n_t} + \\
&Q_{t-1} - \frac{p_t}{P_{t-1}} f f_{t-1}^s + \frac{p_t}{P_{t-1}} \delta t - \tau_t c f_t^s + \frac{c_t}{2} (f t_{t-1}^s - f t_{t-1}^s) - \tau_t (r_k P_{t-1}^H k_{t-1} + d_t) - \tau_t n_t + \tau_t
\end{align*}
\]

\[
\begin{align*}
&- \frac{P_{t-1}^H f f_t^s + p_t f f_t^s + \frac{\phi}{\phi - 1} \left( \frac{S_t f f_t^s}{P_t} - \frac{S_t c f_t^s}{P_t} \right)^2 + \frac{\phi}{\phi - 1} \left( \frac{S_t f f_t^s}{P_t} - \frac{S_t c f_t^s}{P_t} \right)^2}{P_{t-1}^H P_{t-1}^H} \\
&- \frac{S_{t-1} f f_{t-1}^s - f f_{t-1}^s}{P_{t-1}^H P_{t-1}^H}
\end{align*}
\]

\[
(P_t^{\#})^{1-\phi} = \theta P_{t-1}^{1-\phi} + (1 - \theta) (P_t^{\#})^{1-\phi}
\]

\[
P_t = (P_t^H)^{\nu} (P_t^F)^{1-\nu}
\]

\[
P_t^F = S_t P_t^{H^*}
\]
\[ P_t^* = (P_t^{H*})^* \left( \frac{P_t^H}{S_t} \right)^{1-\nu} \]  
(409)

\[ \left( \tilde{P}_t^H \right)^{-\phi} = \theta \left( \tilde{P}_{t-1}^H \right)^{-\phi} + (1 - \theta) \left( \tilde{P}_t^H \right)^{-\phi} \]  
(410)

\[ Q_t = Q_t^* + \psi \left( e^{\frac{b_t + s_t P_t^F - \beta}{\alpha x'} - 1} \right) \]  
(411)

where \[ \Xi_{t+t+k} \equiv \frac{\beta^k c_{t+k}^e}{c_t} P_{t+k} \tau_{t+k}^e \] and \[ \tilde{P}_t^H \equiv \left( \sum_{h=1}^{N} [P_h (b_h)]^{-\phi} \right)^{-\frac{1}{\phi}}. \] Thus, \[ (\tilde{P}_t^H)^{-\phi} \] is a measure of price dispersion.

We thus have 22 equations in 22 endogenous variables, \{\text{\(y_t^H\), \(c_t\), \(c_t^H\), \(c_t^F\), \(n_t, x_t, k_t, \int f_t^h, m_t, P_t^F, P_t, P_t^H, P_t^\#, \tilde{P}_t^H, \tilde{P}_t, \tilde{P}_t^H, \tilde{P}_t^\#, R_t\)}\}_{t=0}^{\infty}. \] This is given technology, \{\(A_k\)}\}_{t=0}^{\infty}, the independently set monetary and fiscal policy instruments, \{\(S_t, \tau_t^e, \tau_t^H, \tau_t^\#, g_t, \tau_t^l, b_t\)}\}_{t=0}^{\infty}, the rest-of-the-world variables, \{\(Q_t^*, P_t^{H*}, c_t^{F*}\)}\}_{t=0}^{\infty}, and initial conditions for the state variables.

### 4.7.5 Decentralized equilibrium transformed (given policy)

We now transform the above equilibrium conditions. In particular, following the related literature, we rewrite them first, by expressing price levels in inflation rates, secondly, by writing the firm’s optimality conditions in recursive form and, thirdly, by introducing a new equation that helps us to compute expected discounted lifetime utility.

**Variables expressed in ratios** We first express prices in rate form. We define 7 new endogenous variables, which are the gross domestic CPI inflation rate \( \Pi_t \equiv \frac{P_t^{H*}}{P_t^{H*-1}} \), the gross foreign CPI inflation rate \( \Pi_t^* \equiv \frac{P_t^{H*}}{P_t^{H*-1}} \), the gross domestic goods inflation rate \( \Pi_t^H \equiv \frac{P_t^H}{P_t^{H-1}} \), the auxiliary variable \( \Theta_t \equiv \frac{P_t^F}{P_t^H} \), the price dispersion index \( \Delta_t \equiv \left( \frac{P_t}{P_t^F} \right)^{-\phi} \), the gross exchange rate depreciation rate \( \epsilon_t \equiv \frac{S_t}{S_{t-1}} \) and the terms of trade \( TT_t \equiv \frac{P_t^F}{P_t^H} = \frac{S_t}{S_{t-1}} \). \[33 \] Thus, in what follows, we use \( \Pi_t, \Pi_t^*, \Pi_t^H, \Theta_t, \Delta_t, \epsilon_t, TT_t \) instead of \( P_t, P_t^{H*}, P_t^H, P_t^\#, \tilde{P}_t, S_t, P_t^F \) respectively.

We also find it convenient to denote \( f_t^{TT} \equiv TT_t^{*} f_t^H \) and then use \( f_t^{TT} \) as the residually determined fiscal policy instrument instead of \( f_t^H \) itself. Thus, \( f_t^{TT} \) is government issued in foreign currency and expressed in domestic currency in real terms. Also, for convenience and comparison with the data, we express fiscal and public finance variables as shares of nominal output, \( P_t^H y_t^H \). In particular, using the definitions above, real government spending, \( g_t \), can be written as \( g_t = s_t^g y_t^H \), real government transfers, \( \tau_t^T \), can be written as \( \tau_t^T = s_t^g y_t^H T_{t+1}^{\nu-1} \), real domestic public debt, \( b_t \), can be written as \( b_t = s_t^g y_t^H TT_{t+1} \). While \( f_t^{TT} \) can be written as \( f_t^{TT} = s_t^l y_t^H \), where \( s_t^g, s_t^l, s_t^b \) and \( s_t^f \) denote respectively the output shares of government spending, government transfers, domestic public debt and foreign public debt (see below for these shares in the data).

\[33 \text{Thus, } \frac{TT_t}{TT_{t-1}} = \frac{TT_t^* f_t^H}{TT_{t-1}^* f_{t-1}^H} = \frac{TT_t}{TT_{t-1}} \]
Equation (401) expressed in recursive form  We now replace equation (401), from the firm’s problem, with an equivalent equation in recursive form. In particular, following Schmitt-Grohé and Uribe (2007), we look for a recursive representation of (388) from the firm’s problem in Appendix 2:

\[
E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P^H_{t+k}}{P_{t+k}} \right]^{-\phi} y_{t+k}^H \left\{ P^H_t - \frac{\phi}{(\phi - 1)} mc_{t+k} P_{t+k} \right\} = 0 \tag{412}
\]

We define two auxiliary endogenous variables:

\[
z^1_t \equiv E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P^H_{t+k}}{P_{t+k}} \right]^{-\phi} \frac{y_{t+k}^H P^H_t}{P_t} \tag{413}
\]

\[
z^2_t \equiv E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P^H_{t+k}}{P_{t+k}} \right]^{-\phi} \frac{y_{t+k}^H mc_{t+k}}{P_t} \tag{414}
\]

Using these two auxiliary variables, \(z^1_t\), \(z^2_t\), and equation (412), we come up with two new equations which enter the dynamic system and allows a recursive representation of (412).

Thus, we have replaced equation (401) with its recursive representation:

\[
z^1_t = \frac{\phi}{(\phi - 1)} z^2_t \tag{415}
\]

where:

\[
z^1_t = \Theta_t^{1-\phi} y_t T_t^{\tau^e_t} + \beta \theta E_t c^e_{t+1} \frac{1 + \tau^e_t}{1 + \tau^c_t} (\Theta_t) \frac{\theta}{\Theta_{t+1}} \left( \frac{1}{\Pi^e_{t+1}} \right)^{1-\phi} z^1_{t+1} \tag{416}
\]

\[
z^2_t = \Theta_t^{-\phi} y_t mc_t + \beta \theta E_t \frac{c^e_{t+1}}{c^e_t} \frac{1 + \tau^e_t}{1 + \tau^c_t} (\Theta_t) \frac{\theta}{\Theta_{t+1}} \left( \frac{1}{\Pi^e_{t+1}} \right)^{-\phi} z^2_{t+1} \tag{417}
\]

Lifetime utility written as a first-order dynamic equation  Since we want to compute social welfare, we follow Schmitt-Grohé and Uribe (2007) by defining a new endogenous variable, \(V_t\), whose motion is given by:

\[
V_t = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi_n \frac{m_t^{1+\phi}}{1+\phi} + \chi_m m_t^{1-\mu} + \chi_g \frac{s_t^g y_t^H}{1-\zeta} + \beta E_t V_{t+1} \tag{418}
\]

where \(V_t\) is household’s expected discounted lifetime utility at time \(t\).

Thus, in what follows, we add equation (418) and the new variable \(V_t\) to the equilibrium system.

Equations of the transformed DE  Then, the transformed DE is summarized by the following 25 equations:

\[
V_t = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi_n \frac{m_t^{1+\phi}}{1+\phi} + \chi_m m_t^{1-\mu} + \chi_g \frac{s_t^g y_t^H}{1-\zeta} + \beta E_t V_{t+1} \tag{419}
\]

\[
\beta E_t \frac{1}{1+\tau_t} \frac{1}{\Pi^e_{t+1}} = c^e_t \tag{420}
\]
\begin{equation}
\beta E_t \, c_t^{1-\sigma} \left( \frac{1}{1 + \tau_{t+1}} \right) Q_t \frac{T_t^{\nu' + \nu - 1}}{\Pi_t^{\nu}} \left[ 1 + \phi^\nu \left( T_t^{\nu' + \nu - 1} f_t^h - T T_t^{\nu' + \nu - 1} f_t^h \right) \right] = c_t^{-\sigma} \left( \frac{1}{1 + \tau_{t+1}} \right) Q_t \frac{T_t^{\nu' + \nu - 1}}{\Pi_t^{\nu}} \left[ 1 + \phi^\nu \left( T_t^{\nu' + \nu - 1} f_t^h - T T_t^{\nu' + \nu - 1} f_t^h \right) \right]
\end{equation}

\begin{equation}
\beta c_t^{-\sigma} T_t^{\nu' - 1} \left\{ 1 - \delta - \frac{2}{\nu} \left( \frac{k_{t+1}}{k_t} - 1 \right)^2 + \xi \left( \frac{k_{t+1}}{k_t} - 1 \right) \frac{k_{t+1}}{k_t} + (1 - \tau_{t+1}^k) \right\} = c_t^{-\sigma} T_t^{\nu' - 1} \frac{1}{(1 + \tau_{t+1})} \left[ 1 - \xi \left( \frac{k_t}{k_{t-1}} - 1 \right) \right]
\end{equation}

\begin{equation}
\chi_m m_t^{-\mu} = c_t^{-\sigma} \left( \frac{1}{1 + \tau_{t+1}} \right) - \beta E_t c_t^{-\sigma} \left( \frac{1}{1 + \tau_{t+1}} \right) \Pi_t^{\nu}
\end{equation}

\begin{equation}
\chi n_t^y = (1 - \tau_{t+1}^n) w_t c_t^{-\sigma} \left( \frac{1}{1 + \tau_{t+1}} \right)
\end{equation}

\begin{equation}
\frac{c_t^F}{c_t^H} = \nu \frac{1}{1 - \nu} T T_t
\end{equation}

\begin{equation}
k_t = (1 - \delta) k_{t-1} + x_t - \frac{\xi}{2} \left( \frac{k_t}{k_{t-1}} - 1 \right)^2 k_{t-1}
\end{equation}

\begin{equation}
c_t = \left( c_t^H \right)^{C_t^F} \left( \frac{1}{\nu} \right)^{1-\nu} (1 - \nu)^{1-\nu}
\end{equation}

\begin{equation}
w_t = mc_t (1 - a) A_t k_{t-1}^{a-1} n_t^{-a}
\end{equation}

\begin{equation}
\frac{1}{T T_t^{\nu' - 1}} = mc_t a A_t k_{t-1}^{a-1} n_t^{-a}
\end{equation}

\begin{equation}
d_t = \frac{1}{T T_t^{\nu' - 1}} y_t^H - \frac{1}{T T_t^{\nu' - 1}} k_t^{1-\nu} k_{t-1} - w_t n_t
\end{equation}

\begin{equation}
z_t^1 = \frac{\phi}{(\phi - 1)} z_t^2
\end{equation}

\begin{equation}
y_t^H = \frac{1}{\Delta_t} A_t k_{t-1}^{1-a}
\end{equation}

\begin{equation}
s_t^b y_t T T_t^{\nu - 1} + m_t + T T_t^{\nu + 1} - \frac{f_t^h}{T T_t^{\nu + 1}} = \frac{\phi^\nu}{2} \left( T T_t^{\nu' + \nu - 1} - T T_t^{\nu' + \nu - 1} f_t^h \right)^2
\end{equation}

\begin{equation}
+ R_{t-1} \frac{1}{\Pi_t} \left( s_t^b y_t T T_t^{\nu - 1} + m_t + \frac{Q_{t-1} T T_t^{\nu + 1} - \frac{f_t^h}{T T_t^{\nu + 1}}}{\Pi_t} + \frac{1}{T T_t^{\nu' - 1}} \right)
\end{equation}

\begin{equation}
- \tau_t \left( \frac{1}{T T_t^{\nu' - 1}} c_t^H + T T_t^{\nu' - 1} c_t^F \right) = -\tau_t \left( \frac{1}{T T_t^{\nu' - 1}} c_t^H + T T_t^{\nu' - 1} c_t^F \right)
\end{equation}

\begin{equation}
y_t^H = c_t^H + x_t + s_t^b y_t^H + e_t^F
\end{equation}

\begin{equation}
T T_t^{\nu + 1} \left( \frac{f_t^h}{T T_t^{\nu + 1}} - f_t^h \right) = \left( T T_t^{\nu' + \nu - 1} - f_t^h \right)
\end{equation}

\begin{equation}
+ \frac{\phi^\nu}{2} \left( T T_t^{\nu' + \nu - 1} f_t^h - T T_t^{\nu' + \nu - 1} f_t^h \right)^2 + \phi^\nu \left( T T_t^{\nu + \nu - 1} - T T_t^{\nu + \nu - 1} \right)^2
\end{equation}

\begin{equation}
\left( \Pi_t^H \right)^{1-\phi} = \theta + (1 - \theta) \left( T T_t^{\nu} \right)^{1-\phi}
\end{equation}

\begin{equation}
\frac{\Pi_t}{\Pi_t^H} = \left( T T_t^{\nu} \right)^{1-\nu}
\end{equation}
We thus have 25 equations in 25 endogenous variables, \( \{V_t, y_t^H, c_t, c_t^F, n_t, x_t, k_t, f_t^h, m_t, TT_t, \Pi_t, \Pi_t^H, \Theta_t, \Delta_t, w_t, mca_t, d_t, r_t^F, Q_t, f_t^TT, \Pi_t^*, z_1^T, z_2^T, R_t \}_{t=0}^{\infty} \). This is given technology, \( \{A_t\}_{t=0}^{\infty} \), the independently set policy instruments, \( \{\epsilon_t, \tau_t^F, \tau_t^H, s_t^F, s_t^H\}_{t=0}^{\infty} \), the rest-of-the-world variables, \( \{Q_t, \Pi_t^H, c_t^F\}_{t=0}^{\infty} \), and initial conditions for the state variables.
5 Fiscal consolidation and its cross-country effects. Is there a conflict of interests?

5.1 Introduction

Most eurozone periphery countries are in a debt crisis. In view of rising sovereign risk premia and solvency concerns, these countries have been forced to take highly restrictive fiscal policy measures which have dampened demand in the short term. It is thus not surprising that fiscal consolidation has been one of the most debated policy areas over the past years. The controversy has been intensified by the lasting recession experienced by these countries. On the other hand, fiscal policy in eurozone center countries, like Germany, has been relatively neutral. Nevertheless, the recession in the crisis countries has also started to affect the German economy, which is another reminder of the importance of spillovers in an integrated area like the euro area.

In this paper, we study how public debt consolidation in a country with high debt and sovereign premia affects welfare in other countries with solid public finances. In particular, we study how public debt consolidation in a country like Italy affects welfare in a country like Germany and how these cross-border effects depend on the fiscal policy mix chosen to bring public debt down.

The setup is a New Keynesian world economy DSGE model consisting of two heterogeneous countries. An international asset allows agents in one country to borrow from, or lend to, agents in the other country. Our modelling implies that, as it is the case in the data, the home country (Germany) is a net lender and the other country is a net borrower (Italy) in the international asset market. This is driven by the assumption that agents differ in their degree of impatience. Namely, borrowers (the Italians) are less patient than lenders (the Germans). Being in a monetary union, there is a single monetary policy. On the other hand, the two countries are free to follow independent fiscal policies. Following most of the literature on debt consolidation, we assume that policy is conducted via simple and implementable feedback rules, meaning that the single monetary policy is conducted via a standard Taylor rule, while national fiscal policy instruments are allowed to respond to the gap between actual public debt and target public debt as shares of output, as well as to the gap between actual and target output. We experiment with various debt targets depending on whether national policymakers aim just to stabilize the economy around its status quo (defined as the solution consistent with the recent public finance data), or whether they also want to move the economy to a new reformed long run (defined as a solution with lower public debt and without sovereign premia). Since we do not want our results to be driven by ad hoc differences in feedback policy coefficients across different rules, we compute optimized feedback policy rules when the welfare criterion is the weighted average of households’ expected lifetime utility in the two countries.

We solve the model numerically employing commonly used parameter values and policy data from Germany (the home country) and Italy (the foreign country). After we check that the model can mimic the key empirical characteristics of the two countries over the recent euro years, we use it as a point of departure to study the dynamic evolution of endogenous variables in response to policy reforms, focusing on debt consolidation in the high-debt country, Italy. Adopting the methodology of Schmitt-Grohé and Uribe (2004 and 2007), we compute the welfare-maximizing values of feedback policy coefficients by taking a second-order approximation to both

34 This is in particularly true in Cyprus, Greece, Italy, Portugal and Spain. See e.g. EMU-Public Finances of the European Commission (2013) and the EEAG Report of CESifo (2013).
35 See e.g. EMU-Public Finances of the European Commission (2013) and the EEAG Report of CESifo (2013).
36 See e.g. the EEAG Report of CESifo (2013).
the equilibrium conditions and the welfare criterion.

Our main results are as follows. First, fiscal consolidation in the high-debt country (Italy) benefits the country with solid public finances (Germany) over all time horizons. By constrast, in Italy itself, namely the country that takes the consolidation measures, such a policy is productive only if its citizens are relatively far-sighted. Thus, fiscal consolidation comes at a short- and medium-term pain in the country that undergoes it.

Second, the higher the say of Germany in policy setting, the stronger the fiscal consolidation in Italy should be. Also, as expected, the higher the say of Germany, the better off becomes Germany and the worse off becomes Italy.

These two first results combined imply that fiscal consolidation in a high debt country is a common interest over long horizons only. By contrast, in shorter horizons, there is a conflict of national interests. All this holds irrespectively of the fiscal policy mix used.

Third, the least distorting fiscal policy mix from the point of view of both countries is that Italy cuts public spending during the early phase of fiscal pain and, once its public debt has been reduced, uses the fiscal space created to cut labor or capital taxes which appear to be particularly distorting. Note that expectations of cuts in labor or capital taxes in the future, once debt consolidation has been achieved, play a key role. Use of public spending is also recommended in Germany, where the policy aim is cyclical stabilization only. In other words, using tax rates for debt reduction and/or shock stabilization is a bad idea from the viewpoint of both countries.

Our paper is related to two literatures. First, it is related to the literature on how monetary and fiscal policy instruments react, or should react, to the business cycle. Second, it is related to the literature on fiscal consolidation that usually compares spending cuts versus tax rises needed for debt reduction. Nevertheless, as far as we know, there have not been any previous attempts to welfare rank a rich menu of tax-spending policy instruments in a New Keynesian DSGE model consisting of two interacting heterogeneous countries and study the cross-border implications of various consolidation measures taken by the high-debt country.

The rest of the paper is organized as follows. Section 2 presents the model. The status quo solution, using data from Germany and Italy, is in section 3. Section 4 explains our policy experiments. Results are in section 5. Section 6 closes the paper.

5.2 Model

This section sets up a New Keynesian world economy DSGE model consisting of two heterogeneous countries forming a monetary union.

5.2.1 Description of the model

Two heterogeneous countries form a closed system. An international asset allows agents in one country to borrow from, or lend to, agents in the other country. International borrowing, or lending, can only take place through a financial intermediary or an international bank (as Curdia and Woodford, 2009 and 2010, point out, this can be thought as a financial friction). The bank is located in the domestic country so any profits made by it are distributed to the citizens of the domestic economy.


See Okano (2014) for a recent New Keynesian currency union model consisting of two countries that provides a good review of the related literature going back to Gali and Monacelli (2005, 2008).
Our modelling implies that one country is a net lender and the other is a net borrower in the international asset market. This systematic difference in behavior is driven by the assumption that agents in the two countries differ in their impatience to consume or, equivalently, in their time preference rates. In particular, the time preference rate of lenders is higher than borrowers’ or, equivalently, borrowers are more impatient than lenders (this is similar to the modelling in e.g. Curdia and Woodford, 2009 and 2010).\textsuperscript{40} We like to think of the lender country as, say, Germany and the borrower country as, say, Italy.

On other dimensions, the model is a rather standard New Keynesian model. Each country produces an array of differentiated goods and, in both countries, firms act monopolistically facing Calvo-type nominal fixities. Nominal fixities can give a real role to monetary policy, at least in the transition path. We assume a single monetary policy but independent national fiscal policies.

In particular, the home economy is composed of $N$ identical households indexed by $i = 1, 2, ..., N$, of $N$ firms indexed by $h = 1, 2, ..., N$, each one of them producing a differentiated domestically produced tradable good, as well as of monetary and fiscal authorities. Similarly, in the foreign economy, where there are $N^*$ identical households indexed by $i^* = 1, 2, ..., N^*$, and $N^*$ differentiated firms indexed by $f = 1, 2, ..., N^*$. Population in both countries, $N$ and $N^*$, is constant over time. We assume for simplicity that the two countries are of equal size, $N = N^*$.

Below, we present the domestic country. The foreign country will be symmetric except explicitly said. A star will denote the counterpart of a variable or a parameter in the foreign country.

5.2.2 Households

This subsection presents household $i$ in the domestic country.

Consumption bundles The quantity of variety $h$ produced at home by domestic firm $h$ and consumed by domestic household $i$ is denoted as $c^H_{i,t}(h)$. Using a Dixit-Stiglitz aggregator, the composite of domestic goods consumed by each household $i$, $c^H_{i,t}$, is given by:\textsuperscript{41}

\[
c^H_{i,t} = \left[ \sum_{h=1}^{N} \kappa[c^H_{i,t}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{444}
\]

where $\phi > 0$ is the elasticity of substitution across goods produced in the domestic country and $\kappa = 1/N$ is a weight chosen to avoid scale effects in equilibrium.

Similarly, the quantity of imported variety $f$ produced abroad by foreign firm $f$ and consumed by domestic household $i$ is denoted as $c^F_{i,t}(f)$. Using a Dixit-Stiglitz aggregator, the composite of imported goods consumed by each household $i$, $c^F_{i,t}$, is given by:

\[
c^F_{i,t} = \left[ \sum_{f=1}^{N^*} \kappa[c^F_{i,t}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{445}
\]

\textsuperscript{40}In Curdia and Woodford (2009, 2010), the systematic difference between lenders and borrowers is driven by the assumption that the marginal utility of consumption is higher for the borrower than for the lender. Thus, in both Curdia and Woodford (2009, 2010) and in our paper, agents differ in their preferences. The difference is that in Curdia and Woodford (2009, 2010) agents differ in their utility functions, while in our paper agents differ in their time preference rates. On the other hand, see e.g. Becker and Mulligan (1997) for the endogenous determination of the time preference rate depending on income, education, effort, religion, etc. See also Nelson et al. (2008) for a model calibrated to the US, where the time preference rate depends on consumption.

\textsuperscript{41}As in e.g. Blanchard and Giavazzi (2003), we find it more convenient to work with summations rather than with integrals.
In turn, having defined \( c_{H,t} \) and \( c_{F,t} \), household \( i \)'s consumption bundle, \( c_{i,t} \), is:

\[
 c_{i,t} = (c_{H,t}^\nu (c_{F,t}^{1-\nu})^{1-\nu}/\nu^\nu(1-\nu)^{1-\nu}
\]

where \( \nu \) is the degree of preference for domestic goods (if \( \nu > 1/2 \), there is a home bias).

**Consumption expenditure, prices and terms of trade**  Household \( i \)'s total consumption expenditure is:

\[
P_t c_{i,t} = P_{t}^{H} c_{i,t}^{H} + P_{t}^{F} c_{i,t}^{F}
\]

where \( P_t \) is the consumer price index (CPI), \( P_{t}^{H} \) is the price index of home tradables, and \( P_{t}^{F} \) is the price index of foreign tradables (expressed in domestic currency).

Each household's total expenditure on home goods and foreign goods are respectively:

\[
P_{t}^{H} c_{i,t}^{H} = \sum_{h=1}^{N} \kappa P_{t}^{H}(h)c_{i,t}^{H}(h)
\]

\[
P_{t}^{F} c_{i,t}^{F} = \sum_{f=1}^{N} \kappa P_{t}^{F}(f)c_{i,t}^{F}(f)
\]

where \( P_{t}^{H}(h) \) is the price of variety \( h \) produced at home and \( P_{t}^{F}(f) \) is the price of variety \( f \) produced abroad, both denominated in domestic currency.

We assume that the law of one price holds meaning that each tradable good sells at the same price at home and abroad. Thus, \( P_{t}^{F}(f) = S_{t} P_{t}^{H*}(f) \), where \( S_{t} \) is the nominal exchange rate (where an increase in \( S_{t} \) implies a depreciation) and \( P_{t}^{H*}(f) \) is the price of variety \( f \) produced abroad denominated in foreign currency. A star denotes the counterpart of a variable or a parameter in the rest-of-the-world. Note that the terms of trade are defined as \( P_{t}^{F}/P_{t}^{H} \), while the real exchange rate is defined as \( S_{t}\).  

**Household’s problem**  Each household \( i \) acts competitively to maximize expected discounted lifetime utility, \( V_0 \):

\[
 V_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t},n_{i,t},m_{i,t},g_t)
\]

where \( c_{i,t} \) is \( i \)'s consumption bundle as defined above, \( n_{i,t} \) is \( i \)'s hours of work, \( m_{i,t} \) is \( i \)'s real money holdings, \( g_t \) is per capita public spending, \( 0 < \beta < 1 \) is the time discount rate, and \( E_0 \) is the rational expectations operator conditional on the information set.

The period utility function is assumed to be of the form (see also e.g. Gali, 2008):

\[
u_{i,t} (c_{i,t},n_{i,t},m_{i,t},g_t) = c_{i,t}^{1-\sigma} - \chi_n^{n_{i,t}^{1+\varphi}} + \chi_m^{m_{i,t}^{1-\mu}} + \chi_g^{g_t^{1-\zeta}}
\]

where \( \chi_n, \chi_m, \chi_g, \sigma, \varphi, \mu, \zeta \) are preference parameters. Thus, \( \sigma \) is a coefficient of intertemporal substitution and \( \varphi \) is the inverse of Frisch labour elasticity.

The period budget constraint of household \( i \) written in real terms is:
\[(1 + \tau_t^i) \left[ \frac{P^H_i}{P_t} e^H_t + \frac{P^F_i}{P_t} e^F_t \right] + \frac{P^H_i}{P_t} x_{i,t} + b_{i,t} + m_{i,t} + S_i P^*_t f^h_{i,t} + \frac{\phi^h}{2} \left( \frac{S_i P^*_t f^h_{i,t} - SP^*}{P} \right)^2 = \]

\[= (1 - \tau_t^h) \left[ \frac{P^H_i}{P_t} k_{i,t-1} + \tilde{w}_{i,t} \right] + (1 - \tau_t^h) w_t n_{i,t} + R_{i,t-1} \frac{P_{i,t-1}}{P_t} b_{i,t-1} + \]

\[- \tau_t^1 f_{i,t-1} + \pi_{i,t} \]

where \( x_{i,t} \) is \( i \)'s domestic investment, \( b_{i,t} \) is the real value of \( i \)'s end-of-period domestic government bonds, \( m_{i,t} \) is \( i \)'s end-of-period real domestic money holdings, \( f^h_{i,t} \) is the real value of \( i \)'s end-of-period internationally traded assets denominated in foreign currency, \( \tau_t^h \) denotes the real return to the beginning-of-period domestic capital, \( k_{i,t-1} \), \( \tilde{w}_{i,t} \) denotes \( i \)'s real dividends received by domestic firms, \( w_t \) is the real wage rate, \( R_{i,t-1} \geq 1 \) denotes the gross nominal return to domestic government bonds between \( t-1 \) and \( t \), \( Q_{i,t-1} \geq 1 \) denotes the gross nominal return to international assets between \( t-1 \) and \( t \), \( \tau_t^1 \) are real lump-sum taxes/transfers to each household, \( \pi_{i,t} \) is profits distributed to the domestic household by the financial intermediary (see below) and \( 0 \leq \tau_t^1, \tau_t^h, \tau_t^p \leq 1 \) are tax rates on consumption, capital income and labour income respectively. Small letters denote real values, namely, \( m_{i,t} = \frac{M_{i,t}}{P_t}, \quad b_{i,t} = \frac{B_{i,t}}{P_t}, \quad f^h_{i,t} = \frac{P^h_{i,t}}{P_t}, \quad w_t = \frac{W_t}{P_t}, \quad \tilde{w}_{i,t} = \frac{\tilde{W}_{i,t}}{P_t}, \quad \tau_t^1 = \frac{\tau_{i,t}}{P_t}, \)

where capital letters denote nominal values. The parameter \( \phi^h \geq 0 \) measures transaction costs related to foreign assets as a deviation from their long-run value, \( f^h_{i,t} \).

The law of motion of physical capital for each household \( i \) is:

\[ k_{i,t} = (1 - \delta) k_{i,t-1} + x_{i,t} - \frac{\xi}{2} \left( \frac{k_{i,t}}{k_{i,t-1}} - 1 \right)^2 k_{i,t-1} \]

where \( 0 < \delta < 1 \) is the depreciation rate of capital and \( \xi \geq 0 \) is a parameter capturing adjustment costs related to physical capital.

Each household \( i \) acts competitively taking prices and policy as given. Details, the first-order conditions and implications for price bundles are in the Appendix (see Appendix 1).

### 5.2.3 Firms

Each domestic firm \( h \) produces a differentiated good of variety \( h \) under monopolistic competition and facing Calvo-type nominal fixities.

#### Demand for firm’s product

Household’s problem above implies that the demand for firm \( h \)'s product is:

\[ y^H_t(h) = e_t^H(h) + x_t(h) + g_t(h) + e_t^F(h) = \left( \frac{P^H_t(h)}{P^H_t} \right)^{-\phi} y^H_t \]

where, demand for firm \( h \)'s product, \( y^H_t(h) \), comes from domestic households’ consumption and investment, \( e_t^H(h) \) and \( x_t(h) \), where \( e_t^H(h) = \sum_{i=1}^N e_{i,t}^H(h) \) and \( x_t(h) = \sum_{i=1}^N x_{i,t}(h) \), from the domestic government, \( g_t(h) \), and from foreign households’ consumption, \( e_t^F(h) = \sum_{i=1}^{N_x} e_{i,t}^F(h) \).

---

42 These costs are not important to the main results but help model the data.
Firm’s problem  Nominal profits of firm $h$ are defined as:

$$\tilde{\Omega}_t(h) \equiv P^H_t(h)y^H_t(h) - r^k_t P^H_t(h)k_{t-1}(h) - W_t n_t(h)$$  \hspace{1cm} (455)

This is maximized subject to the demand function above and the production function:

$$y^H_t(h) = A_t(k_{t-1}(h))^{\alpha}[n_t(h)]^{1-\alpha}$$  \hspace{1cm} (456)

where $A_t$ is an exogenous stochastic TFP process whose motion is defined below.

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In particular, in each period, each firm $h$ faces an exogenous probability $\theta$ of not being able to reset its price. A firm $h$, which is able to reset its price at time $t$, chooses its price $P^H_t(h)$ to maximize the sum of discounted expected nominal profits for the next $k$ periods in which it may have to keep its price fixed. This objective is given by:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k+1} \tilde{\Omega}_{t+k}(h) = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P^H_t(h)y^H_{t+k}(h) - \Psi_{t+k}(y^H_{t+k}(h)) \right\}$$

where $\Xi_{t,t+k}$ is a discount factor taken as given by the firm, $y^H_{t+k}(h) = \left[ \frac{P^H_t(h)}{P^H_{t+k}} \right]^{-\phi} y^H_{t+k}$ and $\Psi_{t}(\cdot)$ is the minimum nominal cost function for producing $y^H_t(h)$ at $t$ so that $\Psi_{t}(\cdot)$ is the associated nominal marginal cost. Details for the firm’s problem and first-order conditions are in the Appendix (see Appendix 2).

5.2.4 Government budget constraint

The period budget constraint of the consolidated government sector expressed in real terms and aggregate quantities is:

$$b_t + \sum_{i=1}^{N} c^H_t \equiv R_t - \frac{P^P_t}{P^H_t} b_{t-1} + \frac{S^P_t}{P^H_t} f^P_t \equiv \frac{P^H_t}{P^H_t} y_t g_t - \frac{P^H_t}{P^H_t} c^H_t + \frac{P^P_t}{P^H_t} c_t^H - \frac{P^H_t}{P^H_t} (r^H_t P^H_t k_{t-1} + \bar{\omega}_1 t) - \frac{P^H_t}{P^H_t} n_t - \frac{P^H_t}{P^H_t} t_t$$  \hspace{1cm} (457)

where $b_t$ is the end-of-period domestic real public debt, $f^P_t$ is the end-of-period foreign real public debt expressed in foreign prices and $m_t$ is the end-of-period stock of real money balances. Thus, we use $c^H_t \equiv \sum_{i=1}^{N} c^H_{t,i}$, $c_t^P \equiv \sum_{i=1}^{N} c_{t,i}$, $k_{t-1} \equiv \sum_{i=1}^{N} k_{t-1,i}$, $\bar{\omega}_1 t \equiv \sum_{i=1}^{N} \bar{\omega}_{1,i}$, $n_t \equiv \sum_{i=1}^{N} n_{t,i}$, $F^P_t \equiv \sum_{i=1}^{N} F^P_{t,i}$, $B_{t-1} \equiv \sum_{i=1}^{N} B_{t-1,i}$ and $T_t \equiv \sum_{i=1}^{N} T_{t,i}$. As above, small letters denote real variables, namely, $b_t \equiv b^P_t$, $m_t \equiv m^P_t$ and $f^P_t \equiv f^P_t$. Also, the government allocates its total expenditure among product varieties $h$ by solving an identical problem with household $i$, so that

$$g_t(h) = \left[ \frac{P^P_t(h)}{P^H_t} \right]^{-\phi} g_t(h)$$

The parameter $\phi \geq 0$ captures transaction costs similar to those of the household.

In each period, one of the fiscal policy instruments $(\tau^P_t, \tau^H_t, \tau^m_t, g_t, \tau^k_t, b_t, f^P_t)$ has to follow residually to satisfy the government budget constraint. We assume that the residual instrument is the end of period total public debt. That is, if we define total nominal public debt in the domestic country as $D_t \equiv B_t + S^P_t F^P_t$, so that in real terms $d_t \equiv b_t + \frac{S^P_t}{P^H_t} f^P_t$, we then have

$$b_t \equiv \lambda_t d_t$$

and

$$S^P_t \equiv (1 - \lambda_t)d_t$$

where $\lambda_t$ denotes the fraction of domestic public debt in total public debt. Then, the exogenously set policy instruments are $(\tau^P_t, \tau^H_t, \tau^m_t, g_t, \tau^k_t, \lambda_t)$, while $d_t$ follows residually (see below for other cases).
5.2.5 World financial intermediary

We use a popular model of financial frictions (see e.g. Uribe and Yue, 2006). International borrowing or lending takes place through a financial intermediary or a bank. This bank is located in the home country. It borrows from domestic investors at a rate $Q_t$ and lends to foreign agents at a rate $Q_t^*$. At the same time, the bank faces operational costs which are increasing and convex in the amount of the loan.

The real profit of the bank is:

$$\pi_t = Q_t \left[ (f_t^g - f_t^h) - \frac{\psi}{2} (f_t^g - f_t^h)^2 - \bar{f} \right] - Q_t \frac{S_t P_t^*}{P_t} (f_t^h - f_t^g) \tag{458}$$

where $\psi \geq 0$ and $\bar{f} \geq 0$ are parameters. Thus, $\bar{f}$ is a threshold value above which costs emerge. According to our notation, $(f_t^g - f_t^h)$ is net foreign liabilities in the foreign country and $(f_t^h - f_t^g)$ is net foreign assets in the home country. Note that since the bank is located in the home country, its profits are distributed to private agents in the domestic country in a lump-sum fashion (see equation (9) above).

The bank chooses the amount of its loan taking $Q_t$ and $Q_t^*$ as given. Using the equilibrium condition that net borrowing equals net lending, namely, $F_t^g - F_t^h = S_t (F_t^h - F_t^g)$ or equivalently, after dividing by $P_t$, $f_t^g - f_t^h = \frac{S_t P_t^*}{P_t} (f_t^h - f_t^g)$, then the optimality condition of the international bank with respect to the amount of the loan is:

$$Q_t^* = \frac{Q_t}{1 - \psi [(f_t^g - f_t^h) - \bar{f}]} \tag{459}$$

so that $Q_t^* > Q_t$. Thus, borrowers have to pay a sovereign premium.43

5.2.6 Monetary and fiscal policy

To solve the model, we need to specify monetary and fiscal policy.

Single monetary policy rule If we had flexible exchange rates, the exchange rate would be an endogenous variable and the two countries’ nominal interest rates, $R_t$ and $R_t^*$, could be free to follow, say, Taylor rules. Here, to mimic the eurozone regime, we instead assume that only one of the interest rates, say Germany’s, $R_t$, can follow a Taylor rule, while $R_t^*$ is an endogenous variable replacing the exchange rate which becomes an exogenous policy variable (this modelling is similar to that in e.g. Benigno and Benigno, 2008).44

In particular, we assume a single monetary policy rule of the form:

$$\log \left( \frac{R_t}{R} \right) = \phi_x \log \left( \frac{\Pi_t}{\Pi} \right) + \phi_y \log \left( \frac{y_t^H}{y^H} \right) + \phi_x^* \log \left( \frac{\Pi_t^*}{\Pi^*} \right) + \phi_y^* \log \left( \frac{y_t^H}{y^H} \right) \tag{460}$$

where $\phi_x$, $\phi_y$, $\phi_x^*$, $\phi_y^*$ $\geq 0$ are feedback monetary policy coefficients on inflation and output in each country and variables without time subscripts denote deterministic steady state values (defined below).

43Recall that, by contrast, in a single open economy model, in order to avoid the problem of non-stationary dynamics, one has to assume a deviation from the benchmark small open economy setup. A popular deviation has been to use an interest-elastic risk premium. This would produce an equation like (16). See Schmitt-Grohé and Uribe (2003) for a review of this literature.

44For various ways of modelling monetary policy in a monetary union, see also e.g. Dellas and Tavlas (2005) and Collard and Dellas (2006).
National fiscal policy rules  Countries can follow independent fiscal policies. Following Schmitt-Grohé and Uribe (2007) and many others, we focus on simple fiscal rules meaning that the fiscal authorities react to a small number of easily observable macroeconomic indicators. In particular, in each country, we allow the spending-tax policy instruments, namely, government spending as share of output, defined as $s^g_t$, and the tax rates on consumption, capital income and labor income, $\tau^c_t$, $\tau^k_t$ and $\tau^n_t$, to react to the public debt-to-output ratio as deviation from a target, as well as to the output gap, according to the linear rules:

\[ s^g_t - s^g = -\gamma^g_t (l_{t-1} - l) - \gamma^y_g (y^H_t - y^H) \]  
\[ \tau^c_t - \tau^c = \gamma^c_t (l_{t-1} - l) + \gamma^y c (y^H_t - y^H) \]  
\[ \tau^k_t - \tau^k = \gamma^k_t (l_{t-1} - l) + \gamma^y k (y^H_t - y^H) \]  
\[ \tau^n_t - \tau^n = \gamma^n_t (l_{t-1} - l) + \gamma^y n (y^H_t - y^H) \]

where $l_t \equiv \frac{R_t \lambda_t D_t + Q_t \frac{S_{t+1}}{S_t} (1 - \lambda_t) D_t}{P^H y^H}$

where $\gamma^q_t \geq 0$ and $\gamma^q_t \geq 0$ for $q \equiv (g, c, k, n)$ are feedback fiscal policy coefficients on inherited public liabilities and output while variables without time subscripts denote deterministic steady state values (defined below). All other fiscal policy instruments (namely, lump-sum transfers, $\tau^l$, and the fraction of domestic public debt in total public debt, $\lambda$) are set at their data average values (see below).

Fiscal policy in the foreign country is modelled similarly.

5.2.7 Exogenous variables and productivity shocks

We now specify the exogenous variables, $\{A_t, A^*_t, \lambda_t, \tau^l_t, \tau^c_t, \tau^* k_t, \tau^* l_t, \frac{S_{t+1}}{S_t} \}_{t=0}^{\infty}$. Starting with TFP, we assume stochastic $AR(1)$ processes of the form:

\[ \log (A_t) = (1 - \rho^a) \log (A) + \rho^a \log (A_{t-1}) + \epsilon^a_t \]  
\[ \log (A^*_t) = (1 - \rho^a) \log (A^*) + \rho^a \log (A^*_{t-1}) + \epsilon^a_{*t} \]

where $0 < \rho^a, \rho^a < 1$ are persistence parameters, variables without time subscript denote long-run values and $\epsilon^a_t \sim N (0, \sigma^a_\epsilon)$, $\epsilon^a_{*t} \sim N (0, \sigma^a_\epsilon^2)$.

45 We focus on distorting policy instruments, because using lump-sum ones would be like a free lunch.

46 For similar rules, see e.g. Schmitt-Grohé and Uribe (2007), Bi (2010) and Cantore et al. (2012). See also European Commission (2011) for fiscal reaction functions used in practice. Note that we include reaction to output because it is usually practice. In any case, since the magnitude of feedback policy coefficients is chosen optimally, adding more indicators is not restrictive (on the contrary).

47 We could assume that the long-run target of public liabilities, $l$, is time-varying and stochastic. Our qualitative results do not depend on this. For time-varying and stochastic debt targets, see Coenen et al. (2008), Erceg and Linde (2013) and Philippopoulos et al. (2013).
The exogenously set fiscal policy instruments, \( \{r^f_t, \lambda_t, r^s_t, \lambda^*_t\}_{t=0}^{\infty} \), or equivalently, if we express lump-sum transfers as share of output, \( \{s^f_t, \lambda_t, s^s_t, \lambda^*_t\}_{t=0}^{\infty} \), as assumed to be constant and equal to their data average values (see below). Finally, since this is a closed-system currency union model, the exogenous gross rate of exchange rate depreciation, \( \frac{S_{t+1}}{S_t} \), is set at 1 for simplicity.

5.2.8 Equilibrium system (given feedback policy coefficients)

We now combine all the above to present the Decentralized Equilibrium (DE) system which is for any feasible policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) households maximize utility; (ii) a fraction \(1-\theta\) of firms maximize profits by choosing an identical price \(P^#_t\), while a fraction \(\theta\) just set their previous period prices; (iii) the international bank maximizes its profit (iv) all constraints, including the government budget constraint and the balance of payments, are satisfied; (v) all markets clear, including the international asset market.

Solution steps and the final DE system is presented in Appendix 3. It consists of a system of 58 equations in 58 variables, \( \{V_t, y^H_t, c^H_t, c^F_t, n_t, x_t, k_t, f_t, m_t, TT_t, \Pi_t, \Pi^F_t, \Theta_t, \Delta_t, w_t, m^c_t, d_t, r^f_t, f^H_t, \Pi^H_t, z^1_t, z^2_t, \pi_t, Q_t, l_t, V^s_t, y^H_t, c^H_t, c^F_t, c^s_t, n^*_t, x^*_t, k^*_t, f^H_t, m^s_t, \Pi^F_t, \Theta^*_t, \Delta^*_t, w^*_t, m^c^*_t, d^*_t, r^f_t, f^H^*_t, \Pi^H^*_t, z^1^*_t, z^2^*_t, \pi^*_t, Q^*_t, l^*_t, R_t, s^p_t, \tau^f_t, \tau^c_t, \tau^H_t, \tau^s_t, \tau^k_t, \tau^e_t, \tau^m_t, \tau^{s+1}_t\}_{t=0}^{\infty} \). This is given the values of feedback policy coefficients as defined in subsection (2.6), the exogenous variables, \( \{A_t, A^*_t, s^f_t, s^s_t, \lambda_t, \lambda^*_t, \frac{S_{t+1}}{S_t}\}_{t=0}^{\infty} \), as defined in subsection 2.7, and initial conditions for the state variables.

To solve this system, we take a second-order approximation around its steady state (we report that we have also experimented with non-approximate dynamics and they give the same results qualitatively). In the next section, we start with the steady state solution when policy is set as in the data. This "status quo" solution will in turn serve as a point of departure to study various policy reforms.

5.3 Data, parameterization and the status quo solution

This section solves the model numerically using common parameter values in related studies and fiscal data from Germany and Italy over 2001-2011. As we shall see, the model’s steady state solution will resemble the main empirical characteristics of the two countries over the recent euro years. Hence, we call it the "status quo" solution.

Recall that, since policy instruments react to deviations of macroeconomic indicators from their long-run values, feedback policy coefficients do not play any role in the long-run solution. Also recall that money is neutral in the long run so that the monetary and exchange rate policy regime do not matter to the real economy in the long run.

5.3.1 Data and parameterization

The fiscal and public finance data for Germany and Italy are from OECD Statistics and the Eurostat. The time unit is meant to be a year. The baseline parameter values, as well as the values of exogenous policy variables, are summarized in Table 1.

The model’s key parameters, \( \beta, \beta^* \) and \( \psi \), are calibrated to match the interest rate premium and the foreign position of the two countries in the data. In particular, the values of \( \beta \) and \( \beta^* \) follow from the Euler equations in the two countries which, at steady state, simplify to:

48Thus, \( s^f_t \equiv \frac{s^f_t}{y^H_t + y^F_t} \) and \( s^s_t \equiv \frac{s^s_t}{y^H_t + y^F_t} \).
\[ \beta Q = 1 \]  
(468)

\[ \beta^* Q^* = 1 \]  
(469)

so that, since \( Q < Q^* \) in the data, \( \beta > \beta^* \). That is, the Germans are more patient than the Italians.

In turn, from the optimality condition of the bank:

\[ Q^* = \frac{Q}{1 - \psi ([f^{**} - f^{*h}) - f]} \]  
(470)

so that, given data from all variables, the value of \( \psi \) follows (while we set \( f = 0 \)).

We fix the values of all other parameters using commonly used values in related studies. We assume that these parameters are the same across countries, so that the two countries can differ in the degree of patience and fiscal policy only. Interestingly, these two differences will be enough to give us a steady state solution close to the data averages.

Table 1: Baseline parameter values and policy variables

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Home</th>
<th>Foreign</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a, a^* )</td>
<td>0.4</td>
<td>0.4</td>
<td>shares of capital</td>
</tr>
<tr>
<td>( \nu, \nu^* )</td>
<td>0.5</td>
<td>0.5</td>
<td>home goods bias parameters</td>
</tr>
<tr>
<td>( \mu, \mu^* )</td>
<td>3.42</td>
<td>3.42</td>
<td>parameters related to money demand elasticity</td>
</tr>
<tr>
<td>( \delta, \delta^* )</td>
<td>0.06</td>
<td>0.06</td>
<td>capital depreciation rates</td>
</tr>
<tr>
<td>( \phi, \phi^* )</td>
<td>6</td>
<td>6</td>
<td>price elasticities of demand</td>
</tr>
<tr>
<td>( \varphi, \varphi^* )</td>
<td>1</td>
<td>1</td>
<td>the inverse of Frisch labour elasticities</td>
</tr>
<tr>
<td>( \sigma, \sigma^* )</td>
<td>1</td>
<td>1</td>
<td>elasticities of intertemporal substitution</td>
</tr>
<tr>
<td>( \theta, \theta^* )</td>
<td>0.2</td>
<td>0.2</td>
<td>price rigidity parameters</td>
</tr>
<tr>
<td>( \chi_m, \chi_n^{m*} )</td>
<td>0.001</td>
<td>0.001</td>
<td>preference parameter related to real money balances</td>
</tr>
<tr>
<td>( \chi_n, \chi_n^r )</td>
<td>5</td>
<td>5</td>
<td>preference parameter related to work effort</td>
</tr>
<tr>
<td>( \xi, \xi^* )</td>
<td>0.01</td>
<td>0.01</td>
<td>adjustment cost parameter on physical capital</td>
</tr>
<tr>
<td>( \phi^h, \phi^h^* )</td>
<td>0.1</td>
<td>0.1</td>
<td>adjustment cost parameter on foreign public debt</td>
</tr>
<tr>
<td>( \phi^h, \phi^{h*} )</td>
<td>0.1</td>
<td>0.1</td>
<td>adjustment cost parameter on private foreign assets</td>
</tr>
<tr>
<td>( \beta, \beta^* )</td>
<td>0.9709</td>
<td>0.957</td>
<td>time preferences</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.095</td>
<td>-</td>
<td>risk premium parameter</td>
</tr>
<tr>
<td>( f )</td>
<td>0</td>
<td>-</td>
<td>threshold parameter in the borrowing cost function</td>
</tr>
<tr>
<td>( \tau^{c}, \tau^{c*} )</td>
<td>0.19</td>
<td>0.17</td>
<td>consumption tax rate</td>
</tr>
<tr>
<td>( \tau^{h}, \tau^{h*} )</td>
<td>0.21</td>
<td>0.32</td>
<td>capital tax rate</td>
</tr>
<tr>
<td>( s^d, s^{d*} )</td>
<td>0.38</td>
<td>0.42</td>
<td>labour tax rate</td>
</tr>
<tr>
<td>( s^l, s^{l*} )</td>
<td>0.21</td>
<td>0.22</td>
<td>government spending as share of output</td>
</tr>
<tr>
<td>( \lambda, \lambda^* )</td>
<td>-0.18</td>
<td>-0.21</td>
<td>lump-sum taxes</td>
</tr>
<tr>
<td>( \lambda, \lambda^* )</td>
<td>0.52</td>
<td>0.61</td>
<td>share of total public debt held by domestic/foreign private agents</td>
</tr>
<tr>
<td>( \sigma_{\alpha}, \sigma_{\alpha^*} )</td>
<td>0.01</td>
<td>0.01</td>
<td>standard deviation of TFP</td>
</tr>
<tr>
<td>( \rho^c, \rho^{c*} )</td>
<td>0.92</td>
<td>0.92</td>
<td>persistence of TFP</td>
</tr>
</tbody>
</table>
5.3.2 Steady state solution or the "status quo"

Table 2 presents the steady state solution of the model economy in section 2 when we use the parameter values and the policy instruments in Table 1. In Table 2, we also present some key ratios in the German and Italian data whenever available. Notice that most of the solved ratios are close to their actual values. This solution will serve as a point of departure. That is, in what follows, we will depart from this solution to study various policy experiments. We report (and this is shown below) that an exogenous reduction is public debt stimulates output and improves welfare in both countries; this can provide a justification for our fiscal consolidation experiments.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Home Data</th>
<th>Foreign Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u, u^*)</td>
<td>steady-state utility</td>
<td>0.49</td>
<td>-</td>
</tr>
<tr>
<td>(n, n^*)</td>
<td>hours worked</td>
<td>0.29</td>
<td>-</td>
</tr>
<tr>
<td>(w, w^*)</td>
<td>real wage rate</td>
<td>0.94</td>
<td>-</td>
</tr>
<tr>
<td>(r_k, r_k^*)</td>
<td>real return to physical capital</td>
<td>0.11</td>
<td>-</td>
</tr>
<tr>
<td>(Q - Q^*)</td>
<td>interest rate premium</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(\frac{c}{y^{TT_t}}), (\frac{c^<em>}{y^{TT_t^{</em>}}-1})</td>
<td>consumption as share of GDP</td>
<td>0.63</td>
<td>0.58</td>
</tr>
<tr>
<td>(\frac{k}{y^{TT_t}}), (\frac{k^<em>}{y^{TT_t^{</em>}}-1})</td>
<td>physical capital as share of GDP</td>
<td>2.93</td>
<td>-</td>
</tr>
<tr>
<td>(\frac{TT_t}{y^{TT_t}}), (\frac{TT_t^<em>}{y^{TT_t^{</em>}}-1})</td>
<td>private foreign assets as share of GDP</td>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>(\frac{d}{y^{TT_t}}), (\frac{d^<em>}{y^{TT_t^{</em>}}-1})</td>
<td>total public debt as share of GDP</td>
<td>0.71</td>
<td>0.71</td>
</tr>
<tr>
<td>(\frac{TT_t^<em>(1-\lambda)^d}{y^{TT_t^{</em>}}-1} - f^h)</td>
<td>total foreign debt as share of GDP</td>
<td>-0.28</td>
<td>-0.21</td>
</tr>
</tbody>
</table>

5.4 Policy experiments and solution strategy

Before we present results in section 5, we need to define the role of fiscal policy in each country and explain how we compute optimized policy rules.

5.4.1 National fiscal policies

Motivated by the facts discussed in the opening paragraph of the Introduction, we choose to study two types of fiscal action, one for each country.

**Fiscal policy in the home country with solid public finances** Since the home country (Germany) follows a neutral fiscal policy, the role of national policy is only to stabilize the economy against shocks. In other words, the home country does not take any fiscal consolidation measures. It just stabilizes the public debt-to-GDP ratio at its average level. For instance, say that the economy is hit by a temporary adverse shock to TFP as modelled in equations (466)-(467). This, as the impulse response functions can show, leads to a contraction in output and a rise in the public debt-to-output ratio. Then, the policy questions are which tax-spending policy instrument to use over time, and how strong the reaction of those policy instruments to
deviations from targets should be, in order to minimize cyclical volatility. In other words, we depart from, and end up at, the same fiscal position.

In particular, fiscal policy in the home country is defined as: (a) All tax-spending policy instruments remain at the same value (data average value) in both the status quo and the new long run. Note that the new long run differs from the status quo because of fiscal consolidation in the foreign country. (b) During the transition to the new long run, all tax-spending instruments are allowed to react to deviations from policy targets. (c) All the time, the public debt serves as the residually determined public financing instrument.

Fiscal policy in the foreign country with weak public finances  In the foreign country (Italy), the role of policy is twofold: to stabilize the economy against the same shock(s) as above and, at the same time, to improve resource allocation by gradually bringing down the public debt-to-GDP ratio and reducing the associated sovereign premium. In other words, we depart from the status quo solution, but we end up at a reformed steady state with a new fiscal position.

In particular, fiscal policy in the foreign country is defined as: (a) In the new long run, the output share of public debt falls from 110% (which is the average value in the Italian data over the recent period and it was also the value in our status quo solution in subsection 3.2) to the target value of 90% and there are no sovereign premia, $Q = Q^*$. (b) In the new long run, since the public debt has been reduced and fiscal space has been created, fiscal spending can be increased, or one of the tax rates can be reduced, depending on which policy instrument is assumed to follow residually. This is known as the long-term fiscal gain from debt consolidation. (c) During the transition to the new long run, all tax-spending instruments are allowed to react to deviations from policy targets. Given that the long term debt target is lower than in the status quo, this requires lower public spending, and/or higher tax rates, during the transition period. This is known as the short-term fiscal pain of debt consolidation.

Nevertheless, we will also solve for the case in which Italy does take any fiscal consolidation measures (that is, it reacts like Germany by just stabilizing its public debt at its recent historically high level); this will serve as a benchmark to evaluate the merits of fiscal consolidation.

How we model fiscal policy To understand the logic of our results, and following usual practice in related studies, we will start by experimenting with one fiscal instrument at a time. This means that, along the early costly phase, we allow only one of the fiscal policy instruments to react to public debt imbalances and, at the same time, it is the same fiscal policy instrument that adjusts residually in the long-run to close the government budget. Thus, we will start by assuming that the same policy instrument bears the cost of, and reaps the benefit from, debt consolidation.

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49 We choose the target value of 90% simply because this is consistent with evidence provided by e.g. Reinhart and Rogoff (2010) and Checherita-Westphal and Rother (2012) that, in most advanced economies, the adverse effects of public debt arise when it is around 90-100% of GDP. Our main results are not sensitive to this.

50 This is only one out of many possible reforms. Alternatively, we could assume that debt is reduced but there are still differences between domestic and foreign interest rates. Our main results are not sensitive to this.

51 It is recognized that debt consolidation implies a tradeoff between short-term pain and medium-term gain (see e.g. Coenen et al., 2008). During the early phase of the transition, debt consolidation comes at the cost of higher taxes and/or lower public spending. In the medium- and long-run, a reduction in the debt burden allows, other things equal, a cut in tax rates, and/or a rise in public spending. Thus, one has to value the early costs of stabilization vis-a-vis the medium- and long-term benefits from the fiscal space created. It is also recognized that the implications of fiscal reforms, like debt consolidation, depend heavily on the public financing policy instrument used, namely, which policy instrument adjusts endogenously to accommodate the exogenous changes in fiscal policy (see e.g. Leeper et al., 2009). In the case of debt consolidation, such implications are expected to depend both on which policy instrument bears the cost of adjustment in the early period of adjustment and on which policy instrument is expected to reap the benefit, once consolidation has been achieved. Notice that if lump-sum policy instruments were available, the costs of adjustment, as well as the benefits after adjustment has been achieved, would be trivial.
consolidation. In turn, we will experiment with fiscal policy mixes, which means that we can use different fiscal policy instruments in the transition and in the long run.

The rules for fiscal policy instruments are as in subsection 5.2.6 above except that now the targeted values are those of the reformed long-run equilibrium. In all experiments, all other fiscal policy instruments, except the one used for stabilization, remain unchanged and equal to their pre-reform status quo values.

Specifically, we work as follows. We first compare the steady state equilibria with, and without, debt consolidation in the foreign country. In turn, setting, as initial conditions for the state variables, their values from the solution without debt consolidation (in particular, from the status quo solution in subsection 4.3.2), we compute the equilibrium transition path as we travel towards the steady state of the reformed economy. This is for each method of public financing used. The feedback policy coefficients of the instrument(s) used for stabilization along the transition path are chosen optimally. The way we compute optimized feedback policy rules with, or without, debt consolidation is explained in the next subsection.

5.4.2 Optimized policy rules

To make the comparison of different policies meaningful, we compute optimized feedback policy rules, so that results do not depend on ad hoc differences in feedback policy coefficients across different regimes. We start with defining the welfare criterion.

Welfare criterion The welfare criterion is a weighted average of household’s expected discounted lifetime utility in the two countries. In particular, we assume:

\[
W_t = \eta V_t + (1 - \eta) V^*_t
\] (471)

where 0 ≤ η ≤ 1 is the political weight of the home country vis-a-vis the foreign country, while \( V_t \) and \( V^*_t \) are as defined in equation (450) above. We will experiment with various values of η.

Notice that this practically means that national fiscal policies, although they are allowed to differ, are set in a coordinated way. We choose this modelling because, these days, most fiscal policy decisions, especially fiscal consolidation measures, are taken under the advice, or coordination, of the European Union (see EMU-Public Finances of the European Commission, 2013). Alternatively, one could solve a Nash game in national fiscal policies.

How we compute optimized feedback policy rules In choosing feedback policy coefficients optimally, we work in two steps. In the first preliminary step, we search for the ranges of feedback policy coefficients (as defined in equations (461-464) for the home country and analogously for the foreign country), which allow us to get a locally determinate equilibrium (this is what Schmitt-Grohé and Uribe, 2007, call implementable rules). If necessary, these ranges will be further restricted so as to give economically meaningful solutions for the policy instruments (e.g. tax rates less than one and non-negative nominal interest rates). In our search for local determinacy, we experiment with one, or more, policy instruments and one, or more, operating targets at a time.

In the second step, within the determinacy ranges found above, we compute the welfare-maximizing values of feedback policy coefficients (this is what Schmitt-Grohé and Uribe, 2005 and 2007, call optimized policy rules). The welfare criterion is to maximize the conditional welfare of the two households as defined in (28) above, where conditionality refers to the initial conditions chosen; the latter are given by the status quo solution above. To this end, following e.g.
Schmitt-Grohé and Uribe (2004), we take a second-order approximation to both the equilibrium conditions and the welfare criterion. As is known, this is consistent with risk-averse behavior on the part of economic agents and, in addition, it can help us to avoid possible spurious welfare results that may arise when one takes a second-order approximation to the welfare criterion combined with a first-order approximation to the equilibrium conditions (for details, see e.g. Gali, 2008, Benigno and Woodford, 2012, and Okano, 2014).

In other words, we first compute a second-order accurate approximation of both the conditional welfare and the decentralized equilibrium, as functions of feedback policy coefficients and in turn, we use a matlab function (such as fminsearch.m or fminsearchbnd.m) to compute the values of the feedback policy coefficients that maximize this approximation (matlab routines are available upon request). In this exercise, as said above, if necessary, the feedback policy coefficients are restricted to be within some prespecified ranges so as to deliver determinacy and give meaningful values for policy instruments. All this is with, and without, debt consolidation. The case without consolidation serves as a benchmark.

5.5 Results
In this section, we present the main results. We start with the reformed long run and in turn study transition dynamics and welfare over different time horizons.

5.5.1 Steady state utility with debt consolidation
The new reformed steady state is as defined in subsection 5.4.1 above. In other words, fiscal policy in the home country (Germany) remains as in the status quo. Namely, all exogenously set tax-spending instruments are set as in the data, while public debt plays the role of the residual instrument. On the other hand, in the foreign country (Italy), the exogenous debt reduction implies that public spending can increase, or a tax rate can be reduced, residually.

Table 3 reports steady state utility in both countries under various scenarios regarding the residual policy instrument used in Italy. For instance, in the first row of Table 3, the assumption is that it is public spending that takes advantage of debt reduction, in the sense that once the debt burden has been reduced, public spending can increase relative to its value in the status quo solution. In the other rows, the fiscal space is used to finance cuts in one of the three tax rates.

If we recall that, in the status quo solution of Table 2, we had $u = 0.491$ for Germany and $u^* = 0.441$ for Italy, the message is that debt consolidation in the high-debt country is Pareto-efficient in the long run. In other words, long-run utility rises in both countries. Our solution for the rest of the endogenous variables implies that Italy gains thanks to the fiscal space created but also because of higher consumption, exports and more competitive terms of trade. Germany also gains mainly because of higher exports to prospering Italy. Notice also in Table 3 that the highest utility for both countries is achieved when Italy uses capital or labor taxes as the residual fiscal instrument (this is discussed below).

\[^{52}\text{In the new reformed long run, Italy can increase the output share of government spending by 1.8\%, or reduce the consumption tax rate by 3.4\%, or reduce the capital tax rate by 4\% or reduce the labour tax rate by 4.3\%.}\]
5.5.2 Local determinacy

It is well recognized that the interaction between fiscal and monetary policy, and in particular the magnitude of the associated feedback policy coefficients, are crucial to local determinacy (see e.g. Leith and Wren-Lewis, 2008, in a related set up). Before we present transition results, we report that economic policy can guarantee local determinacy when fiscal policy instruments ($s_t^g$, $\tau_t^n$, $\tau_t^k$, $n_t$) react to public liabilities between critical minimum and maximum non-zero values, where these critical values differ across different fiscal policy instruments, and when monetary policy satisfies the so-called Taylor principle, meaning that the single nominal interest rate reacts aggressively to inflation. This is also shown by the results for optimized policy rules below. By contrast, fiscal and monetary policy reaction to the output gap is not found to be crucial to determinacy. Further details regarding ranges of feedback policy coefficients guaranteeing local determinacy are available upon request.

5.5.3 Lifetime utility with debt consolidation

Within the determinacy ranges found, we now compute optimized policy rules and the associated expected discounted lifetime utility. Recall that the home country goes for shock stabilization only (in particular, shocks to TFP), while the foreign country goes for both shock stabilization and debt consolidation. Also recall that we start with the case in which we use the same fiscal policy instrument over time and across countries.\textsuperscript{53}

Results are reported in Table 4. The first column lists the monetary and fiscal policy instruments used in each scenario, the second column reports the optimal reaction of the single nominal interest rate to inflation in the two countries, and the third column reports the optimal reaction of national fiscal policies to national public debt. The last three columns report expected discounted lifetime utility in the whole union, the domestic country (Germany) and the foreign country (Italy) respectively.\textsuperscript{54}

Recall that, under each policy regime, feedback policy coefficients are chosen to maximize the weighted average in (471), where we start by assuming the politically neutral case in which the two countries matter the same, i.e. we set $\eta = 0$. We report that the resulting values of all instruments used are well-defined in all solutions and over all periods, meaning that tax rates are between zero and one and that the nominal interest rate is above the zero bound. Recall also that here we compare results under optimized policy rules so one should not except to see big differences across different policy regimes.

\textsuperscript{53}In particular, as said above in subsection 4.1, Italy and Germany use the same fiscal instrument over the transition, and this instrument is also used by Italy in the long run to close its government budget. In Germany, public debt plays the role of the residual instrument all the time including the long run.

\textsuperscript{54}To compare welfare across regimes, we could also use a flat consumption subsidy that makes the agent indifferent between two regimes (see e.g. Lucas, 1990). The policy message will be the same.
Table 4: Optimized rules and lifetime utility with debt consolidation in Italy ($\eta = 0.5$)

<table>
<thead>
<tr>
<th>Instruments used</th>
<th>Monetary reaction to inflation rates</th>
<th>National fiscal reactions to debt</th>
<th>$E_0V_0$</th>
<th>$E_0V_0$</th>
<th>$E_0V_0^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_t$ $s_t^g$ $s_t^g$</td>
<td>$\phi_s = 1.1168$ $\phi_s^* = 0$</td>
<td>$\gamma_t^g = 0.0296$ $\gamma_t^g = 0.5081$</td>
<td>16.9083</td>
<td>17.4643</td>
<td>16.3523</td>
</tr>
<tr>
<td>$R_t$ $\tau_t^c$ $\tau_t^c$</td>
<td>$\phi_c = 0$ $\phi_c^* = 3$</td>
<td>$\gamma_t^c = 0.3621$ $\gamma_t^c = 0.9016$</td>
<td>17.0125</td>
<td>17.6372</td>
<td>16.3877</td>
</tr>
<tr>
<td>$R_t$ $\tau_t^k$ $\tau_t^k$</td>
<td>$\phi_c = 3$ $\phi_c^* = 0$</td>
<td>$\gamma_t^k = 0.629$ $\gamma_t^k = 2$</td>
<td>17.2065</td>
<td>17.7853</td>
<td>16.6277</td>
</tr>
<tr>
<td>$R_t$ $\tau_t^n$ $\tau_t^n$</td>
<td>$\phi_c = 3$ $\phi_c^* = 3$</td>
<td>$\gamma_t^n = 0.1208$ $\gamma_t^n = 0.1194$</td>
<td>17.0802</td>
<td>17.7727</td>
<td>16.3878</td>
</tr>
</tbody>
</table>

Notes: In all solutions, $R_t \geq 1$, $0 < s_t^g$, $\tau_t^c$, $\tau_t^k$, $\tau_t^n < 1$, at all $t$.

Although we prefer to postpone a detailed interpretation of our results until below when we study policy mixes and various time horizons, it is worth pointing out, at this early stage, two results that remain unchanged throughout the paper. First, in all cases and over all time horizons, welfare in Germany (the patient country) is higher than welfare in Italy (the impatient country). Second, when we are restricted to use the same fiscal policy instrument in both countries and all the time, the capital tax rate scores better than all other fiscal policy instruments and this applies to both countries. This happens because, although fiscal consolidation implies a tradeoff between short-term pain and long-term gain in Italy, expectations of cuts in capital taxes in the future, once fiscal consolidation has been achieved, dominate over any other short-term effects and this is good for both countries since such expectations stimulate investment, growth and exports in both countries.55

The fact that it is expectations of cuts in capital and labor taxes in the future, that play the dominant role in lifetime results, is confirmed when we assume instead that the fiscal space created by fiscal consolidation in Italy is used to increase lump-sum transfers in this country, rather than to reduce distorting taxes, at steady state. In this case, with trivial expected benefits from fiscal consolidation, government spending and consumption taxes score better than capital and labor taxes. Results for this case are in Table 5.

55 This is also the case in a semi-small open economy (see Philippopoulos et al., 2013). By contrast, in a closed economy without sovereign premia, public spending scores the best in terms of expected discounted lifetime utility (see Philippopoulos et al., 2012).
### Table 5: Optimized rules and lifetime utility with debt consolidation in Italy when Italy uses the fiscal space to increase lump-sum transfers ($\eta = 0.5$)

<table>
<thead>
<tr>
<th>instruments used</th>
<th>monetary reaction to inflation</th>
<th>national fiscal reactions to debt</th>
<th>$E_0W_0$</th>
<th>$E_0V_0$</th>
<th>$E_0V_0^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_t$</td>
<td>$s_t^g$</td>
<td>$s_t^g$</td>
<td>$\phi_t = 0.1654$</td>
<td>$\gamma_t^g = 0.0294$</td>
<td>17.1519 17.7780 16.5257</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\phi_t^c = 0.8819$</td>
<td>$\gamma_t^c = 0.6926$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_t$</td>
<td>$\tau_t^c$</td>
<td>$\tau_t^c$</td>
<td>$\phi_t = 3$</td>
<td>$\gamma_t^c = 0.2491$</td>
<td>16.9145 17.4574 16.3715</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\phi_t^c = 0$</td>
<td>$\gamma_t^c = 0.7340$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_t$</td>
<td>$\tau_t^k$</td>
<td>$\tau_t^k$</td>
<td>$\phi_t = 3$</td>
<td>$\gamma_t^k = 0.0645$</td>
<td>16.8765 17.4469 16.3062</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\phi_t^k = 0$</td>
<td>$\gamma_t^k = 1.4165$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_t$</td>
<td>$\tau_t^n$</td>
<td>$\tau_t^n$</td>
<td>$\phi_t = 3$</td>
<td>$\gamma_t^n = 0.1497$</td>
<td>16.7110 17.2070 16.2149</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\phi_t^n = 2.9859$</td>
<td>$\gamma_t^n = 0.1185$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: In all solutions, $R_t \geq 1, 0 < s_t^g, \tau_t^c, \tau_t^k, \tau_t^n < 1$, at all $t$.

### 5.5.4 Using different fiscal instruments over time and across countries

So far, we have studied one fiscal instrument at a time and this was both over time and across countries. Now we allow for policy mixes. In particular, both countries can now use all their national tax-spending policy instruments in the transition period. To the extent that feedback reactions are chosen optimally, this will tell us which fiscal mix is the least distorting along the transition to the new reformed long run. Also, we study what happens when Italy, the country that undergoes fiscal consolidation, uses different policy instruments in the long run once its public debt has been reduced.56

The welfare implications of such mixes, all with debt consolidation in Italy, are reported in Table 6. The first row reports the case in which Italy reduces its labor tax rate once its public debt has been reduced. The next two rows report results when Italy cuts its capital tax rate or its consumption tax rate, while the last row reports results when the fiscal space is used to raise public spending. From the viewpoint of the monetary union as a whole, as well as from the viewpoint of Germany, the highest expected discounted utility is obtained by the mix in the first row. This mix implies that it is better to use public spending only during the early phase (the other feedback policy coefficients are practically zero), and this is irrespectively of whether the fiscal authorities aim at shock stabilization only or at both debt consolidation and shock stabilization, and in turn, once public debt has been stabilized, to cut labor taxes.

56 As said, Germany always uses public debt as the residual instrument in the long run.
5.5.5 Welfare over various time horizons with, and without, debt consolidation

We now study what happens to welfare over various time horizons. This is important because, for several (e.g. political-economy) reasons, economic agents can be short-sighted in which case expectations about future benefits from fiscal consolidation do not matter. Studying various time horizons can also help us to understand the possible conflicts between short-, medium- and long-term effects from debt consolidation.

To save on space, we focus on the case in which we use the best policy mix found, namely the mix in the first row of Table 6 above. Setting the feedback policy coefficients as in the first row of Table 6, the expected discounted utility over various time horizons for the two countries is reported in Table 7. Numbers in parentheses report results without debt consolidation, other things equal. As explained above, without debt consolidation, we again compute optimized feedback policy rules but now the economy starts from, and also returns to, its status quo with transition dynamics driven by temporary shocks only.57

There is a key message from Table 7. Other things equal, debt consolidation in Italy is always good for Germany. By constrast, in Italy, debt consolidation improves welfare only if we are relatively far-sighted. In particular, our results imply that expected discounted utility is higher with debt consolidation only when we care beyond the first 20 periods, where the exact turning year depends on the fiscal policy mix used, in the sense that the more distorting the mix,

57The optimal feedbacks for the case without debt consolidation are $\phi_\pi = 1.3465$, $\phi_{\pi^*} = 1.3176$, $\gamma^g_1 = 0.1411$, $\gamma^c_1 = 0.071$, $\gamma^k_1 = 0.1312$, $\gamma^{*c}_1 = 0.1013$, $\gamma^{*k}_1 = 0.1179$, $\gamma^{*n}_1 = 0$, $\gamma^{k^*}_1 = 0$, $\gamma^{n^*}_1 = 0$. \n
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the slower the fiscal correction should be. Reversing the argument, fiscal consolidation comes at a short-term welfare cost in the country that undergoes it, while the same fiscal correction is all the time beneficial to the other countries with solid public finances. Thus, as it happens with most reforms, the argument for, or against, debt consolidation involves a value judgment.

Table 7: Welfare in Germany and Italy over different time horizons with, and without, debt consolidation in Italy ($\eta = 0.5$)

<table>
<thead>
<tr>
<th></th>
<th>2 periods</th>
<th>4 periods</th>
<th>10 periods</th>
<th>20 periods</th>
<th>lifetime</th>
<th>long run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>1.1437</td>
<td>2.1864</td>
<td>4.8416</td>
<td>8.0814</td>
<td>18.2708</td>
<td>0.502807</td>
</tr>
<tr>
<td></td>
<td>(0.9659)</td>
<td>(1.8754)</td>
<td>(4.2989)</td>
<td>(7.3217)</td>
<td>(16.8587)</td>
<td>(0.490818)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.5791</td>
<td>1.2838</td>
<td>3.4368</td>
<td>6.4492</td>
<td>16.6421</td>
<td>0.517672</td>
</tr>
<tr>
<td></td>
<td>(0.8614)</td>
<td>(1.6479)</td>
<td>(3.6151)</td>
<td>(5.7805)</td>
<td>(10.2491)</td>
<td>(0.490818)</td>
</tr>
</tbody>
</table>

Note: Results without debt consolidation in parentheses.

5.5.6 Does political power matter?

So far, we have restricted ourselves to the "politically correct" case in which the two countries shared equal political power in policy decision making. Thus, we had set the weight $\eta$ in equation (471) at 0.5. Now we examine what happens over the whole range $0 \leq \eta \leq 1$ varies. Recall that the higher is $\eta$, the more Germany matters. Results are reported in Table 8. To save on space, we focus again on the best policy mix found in Table 6, namely, when Italy and Germany use public spending in the transition phase while Italy cuts labor taxes once its fiscal consolidation has been implemented.

The main messages are as follows. First, as expected, the higher the say of Germany, the better off becomes Germany and the worse off becomes Italy. Second, the higher the say of Germany, the stronger the fiscal consolidation in Italy. This is shown by the monotonic positive effect of $\eta$ on the magnitude of the feedback fiscal policy coefficients on public debt in Italy. In particular $\gamma^g_1$ rises, as $\eta$ rises (the other feedback policy coefficients are practically zero).

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58It should be pointed out that the rise in welfare is partly driven by the fact that debt consolidation and elimination of sovereign premia in the reformed long-run equilibrium allow a higher value of the time preference rate than in the pre-reformed long-run solution in section 5.3 (in particular, the calibrated value of $\beta$ was 0.9603 in the status quo solution in section 4.3.2, while it is 0.9709 without premia).
5.6 Concluding remarks

This paper studied fiscal and monetary policy in a New Keynesian model consisting of two heterogeneous countries being part of a monetary union. We used optimized, simple and implementable feedback policy rules for various categories of taxes and public spending, as well as of the common nominal interest rate, in order to study the general equilibrium implications of fiscal consolidation in the high-debt country. A main result is that, although there is a common interest in the long term, there is a conflict of national interests in shorter horizons, and this is irrespectively of the policy mix chosen for debt consolidation and/or shock stabilization in each country. An extension could be to search for unconventional union-wide policies that reduce the short-term pain in the country that undergoes fiscal consolidation without hurting the other country.

<table>
<thead>
<tr>
<th>weight on</th>
<th>optimal national fiscal reactions to debt</th>
<th>$E_0W$</th>
<th>$E_0V_0$</th>
<th>$E_0V_0^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany’s welfare</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta = 0.3$</td>
<td>$\phi_\pi = 1.0708$  $\phi_{\pi^*} = 0.0057$  $\gamma_\pi^g = 0.0295$  $\gamma_\pi^g = 0.6144$  $\gamma_\pi^{ic} = 0$  $\gamma_\pi^{ik} = 0$  $\gamma_\pi^{in} = 0$  $\gamma_\pi^{in} = 0$</td>
<td>17.1342</td>
<td>18.2731</td>
<td>16.6460</td>
</tr>
<tr>
<td>$\eta = 0.4$</td>
<td>$\phi_\pi = 1.0662$  $\phi_{\pi^*} = 0.0091$  $\gamma_\pi^g = 0.0295$  $\gamma_\pi^g = 0.6322$  $\gamma_\pi^{ic} = 0$  $\gamma_\pi^{ik} = 0.0011$  $\gamma_\pi^{in} = 0$  $\gamma_\pi^{in} = 0$</td>
<td>17.2970</td>
<td>18.2756</td>
<td>16.6447</td>
</tr>
<tr>
<td>$\eta = 0.5$</td>
<td>$\phi_\pi = 1.0629$  $\phi_{\pi^*} = 0.0061$  $\gamma_\pi^g = 0.0293$  $\gamma_\pi^g = 0.7068$  $\gamma_\pi^{ic} = 0$  $\gamma_\pi^{ik} = 0.001$  $\gamma_\pi^{in} = 0$  $\gamma_\pi^{in} = 0.0007$</td>
<td>17.4594</td>
<td>18.2768</td>
<td>16.6421</td>
</tr>
<tr>
<td>$\eta = 0.6$</td>
<td>$\phi_\pi = 1.0676$  $\phi_{\pi^*} = 0$  $\gamma_\pi^g = 0.0295$  $\gamma_\pi^g = 0.7554$  $\gamma_\pi^{ic} = 0$  $\gamma_\pi^{ik} = 0$  $\gamma_\pi^{in} = 0$  $\gamma_\pi^{in} = 0$</td>
<td>17.6239</td>
<td>18.2796</td>
<td>16.6403</td>
</tr>
<tr>
<td>$\eta = 0.7$</td>
<td>$\phi_\pi = 1.0648$  $\phi_{\pi^*} = 0$  $\gamma_\pi^g = 0.0295$  $\gamma_\pi^g = 0.8251$  $\gamma_\pi^{ic} = 0$  $\gamma_\pi^{ik} = 0$  $\gamma_\pi^{in} = 0$  $\gamma_\pi^{in} = 0$</td>
<td>17.7881</td>
<td>18.2821</td>
<td>16.6356</td>
</tr>
</tbody>
</table>
References


[48] Leeper E., 2010: Monetary science, fiscal alchemy, mimeo.


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